

EFR summary

Microeconomics, FEB11001X

2022-2023



Lectures 1 to 18

Weeks 1 to 7

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Details

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Microeconomics – IBEB – Lecture 1, week 1

What is Microeconomics?

Microeconomics is a branch of economics that deals with how individual economic entities like people, firms, and organizations make decisions under the conditions of scarcity.

It deals with the study of individual decisions and group behavior in individual market settings.

Scarcity here does not only entail resources such as money but also includes immaterial factors like time and energy.

Thus, for a choice to be economical, the exchange of money is not a prerequisite.

Why study Microeconomics?

Economists assume that the agents act as if they know the laws of economics.

Microeconomics thus, helps in the following way:

- a. Provides you with a predictive power which is useful even if it turns out to be wrong.
- b. Helps you make the right decisions.

How to study Microeconomics?

To think like an economist, the subject provides you with a set of tools that have a common core foundation and that you can later apply to specific problems. The common core helps you to look at human behaviour through cost and benefits and focus on the margin (covered later).

Hence, when we are comparing an economist to a manager, the key difference is that the manager may have a philosophy or a particular way of doing something but does not possess a set of tools to solve a given problem.

What are the central assumptions in economics?

1. People are considered to be selfish.
2. People are considered to be rational, i.e. they will make the best choice given their desires and knowledge.
3. Money is central; it is considered to be a tool of representation that allows us to compare different options.
4. Economics is simply about making cost-benefit analysis.
5. Economics is about efficiency and not equity or moral judgement.
6. Markets (under certain conditions) constitute one of the most powerful ways of achieving efficiency in economics.

Golden Rule 1: Skimming Reality

According to well trained economists the underlying common code of human behaviour is made up of 0s and 1s.

The 0s and 1s represent marginal costs and marginal benefits and the job of an economist is basically to decipher this common code.

Golden Rule 2: The Margin

Average cost/benefit refers to the ratio of the total cost/benefit to the total number of units of activity.

The average cost for undertaking "x" units of activity = total cost of activity / x

Average benefit for undertaking "x" units of activity = total benefit of activity / x

Marginal cost/benefit refers to the change in the total cost due to the change of one unit of activity.

The level of an activity can be increased as long as the marginal benefit is greater than the marginal cost.

Example:

You are considering buying a yearly membership at a recreational club for \$125. Once you pay the cost, you can access as many activities as you want.

| No. of activities | Total benefit | Total cost | Average benefit | Average Cost |
|-------------------|---------------|------------|-----------------|--------------|
| 1 | 120 | 125 | 120 | 125 |
| 2 | 200 | 125 | 100 | 62.5 |
| 3 | 240 | 125 | 80 | 41.66 |
| 4 | 250 | 125 | 62.5 | 31.25 |

Would you purchase the membership?

Computing the marginal cost and benefits we get:

| Marginal Benefit | Marginal Cost |
|------------------|---------------|
| 120 | 125 |
| 80 | 0 |
| 40 | 0 |
| 10 | 0 |

Thus, it is only sensible to buy the membership if you are going to participate in at least 2 activities.

Important concepts in Microeconomics

In order to perform the cost-benefit analysis in the most efficient manner, there are several concepts that are crucial to know.

1. **Reservation Cost:** This refers to the minimum cost at which an agent is willing to do an activity.

2. Implicit costs like the opportunity cost: This refers to the value of the next best alternative(s).
3. Sunk cost: This refers to the costs that have already been incurred, and therefore can be ignored.

Normative and Positive Questions

Normative Questions deal with what “ought to” or should be done in a particular situation.

For example, Students should attend classes on time.

Positive Questions deal with the consequences of specific policies/decisions.

For example, analysis of the new tax policy enforced by the government.

Microeconomics – IBEB – Lecture 2, week 1

What is a market?

A market is a place – physical or immaterial – where the exchange of goods and services takes place. Here, the sellers usually offer their goods and services to buyers in exchange for money.

Example: A second-hand cycle market fair.

The buyers in the market represent the demand side. All things equal (*ceteris paribus*) buyers prefer cheaper goods.

The sellers in the market represent the supply side. All things equal, the sellers prefer to sell goods at the highest possible price.

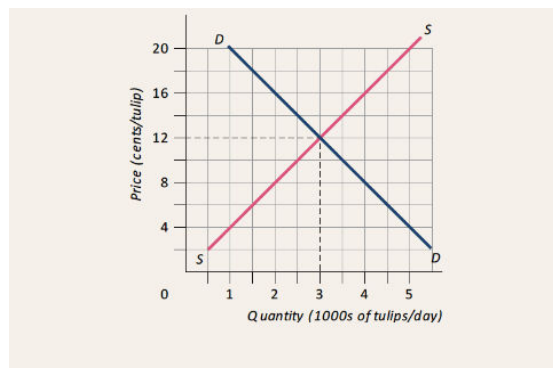
The demand and supply can be represented by lines and curves, and the intersection of these lines provides us with the most efficient output and price level.

Equilibrium

The point at which the demand curve and the supply curve intersect is known as the equilibrium point.

This is the price-quantity combination where the buyers and the sellers are in agreement.

The negative sloping demand curve represents the law of diminishing marginal returns according to which the satisfaction you get from the consumption of each additional unit decreases with the increase in the number of units.



Slide 5; Chapter 2 [presentation slides]

Demand and Supply are not always straight lines but are only used for representation purposes here.

Adjustments in Equilibrium

The market is considered to be inefficient when it is not in equilibrium. This can happen during two situations:

- a. Excess Demand: is when at a given price the buyers demand more goods than the sellers have to offer.
- b. Excess Supply: is when at a given time the buyers demand less goods than the sellers have to offer.



Slide 6; Chapter 2 [presentation slides]

This equilibrium is automatically adjusted through the force of Adam Smith's invisible hand and individual pursuit of self interest. According to this concept, when people act in their own interest, it ultimately benefits the economy. As a result, welfare is also a product of this process as moving towards the equilibrium always makes some agents better off but no agent worse off.

Role of Prices in the Adjustment Process

1. Rationing: This is when the commodities are apportioned to the individuals who are willing to pay the most for them, and hence all bids are not satisfied.
2. Allocation: This is when all bids are satisfied and the commodity is apportioned to those individuals who highly value the transaction.

Here it's important to know that efficiency is different from equity as it does not necessarily tell us which option is "right" but just which is the most optimal for the consumers.

Moving along the Demand and the Supply Curve

When we encounter a change in the quantity or the price of either demand or supply, we move along the consequent curve.

Moving the Demand and the Supply Curve

When we encounter changes in the external factors that affect demand and supply, we move the consequent curve itself.

| Determinants of Demand | Determinants of Supply |
|--------------------------------------|------------------------------------|
| Income | Technology |
| Tastes | Factor prices: labour and capital. |
| Price of substitutes and complements | Expectations |
| Expectations | Number of sellers |
| Number of buyers | Other factors like weather etc. |

Algebra behind the Demand and the Supply curves

Demand: $P = a - bQ$ where $a \geq 0$ and $b > 0$

Supply: $P = c + dQ$ where $d > 0$

Equilibrium point is when the demand equals supply and hence,

$$a - bQ = c + dQ$$

$$Q^* = (a-c)/(c+d); \text{ where } a > c \text{ and } Q^* \text{ is the equilibrium quantity}$$

In order to find equilibrium price (P^*), we plug in the value for Q^* into either demand or supply function.

$$P^* = a - bQ^* = c + dQ^*$$

Taxation

In this course, we focus on the concept of unit tax. This is the sum of money the government needs to receive per unit of the good/service exchanged. Legal incidence identifies the agent who pays the tax to the government whereas economic incidence identifies (non-observable) share of the tax that is borne by the different agents.

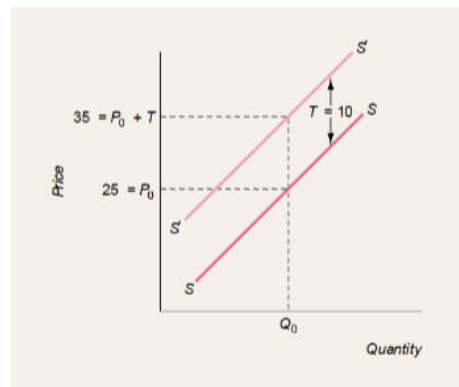
The incidence of tax does not matter when we talk about the economic effect of taxes on the market.

Effect of Taxes

A unit tax t levied on suppliers shifts supply upwards by t units; a unit tax t levied on buyers shifts demand downwards by t units.

TAX PAID BY SUPPLIERS S : $P = c + dQ \rightarrow P = (c + t) + dQ$

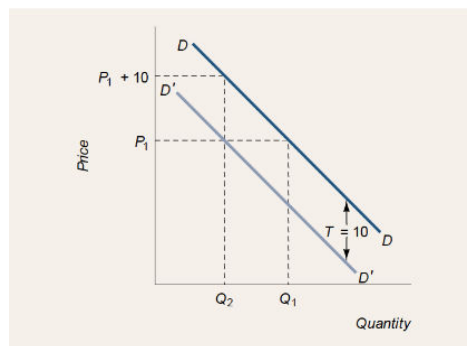
This is a parallel shift upwards because the tax is the same per unit regardless of how many units are purchased.



Slide15; Chapter 2 [presentation slides]

TAX PAID BY BUYERS D : $P = a - bQ \rightarrow P = (a-t) - bQ$

This is a parallel shift downwards because the tax is the same per unit regardless of how many units are purchased.



Slide 14; Chapter 2 [presentation slides]

Through calculations, we can see that the equilibrium quantity is the same whether buyers or sellers pay the tax.

$$Q_t = (a - c - t) / (b + d)$$

Shares of the tax burden

Let (Q^*, P^*) denote equilibrium before tax and (Q^t, P^t) denote equilibrium after tax:

$$\text{Buyers' share of tax} = (P^t - P^*) / t$$

$$\text{Sellers' share of tax} = [P^* - (P^t - t)] / t$$

In order to see if your calculations are correct, check if the sum of the two shares equals 1.

Subsidies

Subsidies are the opposite of tax.

A subsidy given to consumers shifts demand upwards by s units:

$$D : P = a - bQ \rightarrow P = (a + s) - bQ$$

A subsidy given to suppliers shifts supply downwards by s units:

$$S : P = c + dQ \rightarrow P = (c - s) + dQ$$

Rational Consumer Choice Theory

One of the main starting points in Microeconomics is understanding how individuals make choices. Individual purchase decisions, which in turn add up to demand curves, can be explained through rational consumer choice theory.

The above-mentioned theory starts with 3 main assumptions:

1. Consumers' preferences are well-defined and stable.
2. Consumers accept prices as given.
3. Everyone plays by the rules, e.g. no stealing.

The three-step procedure includes the following:

Step 1: Describing all possible consumer choices.

Step 2: Describing consumer preferences.

Step 3: Selecting the preferred option from all the given choices. (can be maximum, or equilibrium)

The Budget Constraint

The combination of two goods, let's say housing, S , and food, F is referred to as a bundle.

The consumption of these goods is given by: $S, F \geq 0$.

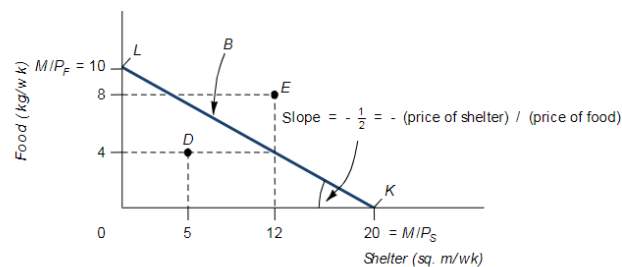
The budget constraint states refer to the set of bundles for which the consumer spends all his income.

Let the prices for F be P_f and the prices for housing be P_s .

The budget constraint is thus: $S * P_s + F * P_f = M$, where M is the income.

Hence, the equation of the line can be written as:

$$F = \frac{M}{P_f} - \frac{P_s}{P_f}S$$
$$(Y = a - bX)$$



Slide 8; Chapter 2 [presentation slides]

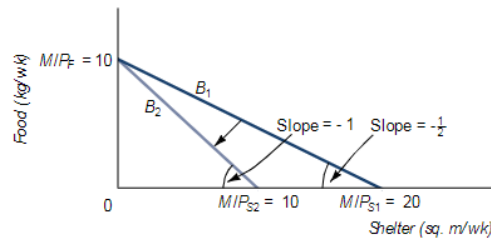
The budget set (also known as feasible/opportunity/affordable budget) is the set of bundles on or below the budget constraint. All points on or below the budget constraint cost less or equal to M , so are therefore affordable.

Slope of the budget line = $-(\text{Price of } S) / (\text{Price of } F) = -P_s / P_f$

In reality, the budget line is not always so simplified but can have holes, gaps or kinks (covered later).

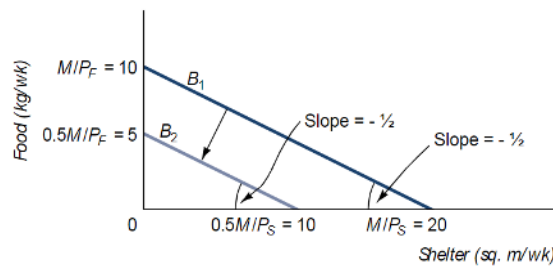
Effects of change in price and income on budget constraints

When the price of one good changes or the prices of both goods change disproportionately, the slope changes. So, for example, if P_S increases, the intercept between the y-axis and the budget constraint remains the same while the budget line rotates inwards towards the origin.



Slide 12; Chapter 4 [presentation slides]

When the prices of both goods change proportionally or the income changes, the slope does not change and the new budget line shifts parallel to the old one. So, for example, if income decreases, the budget line will move towards the origin, parallel to the original line.



Slide 14; Chapter 4 [presentation slides]

Note: for “buy one, take one” deals, we simply need to cut the price of the good in half if we want to represent it in the budget line.

Microeconomics – IBEB – Lecture 3, week 1

Budget Constraints of more than two goods

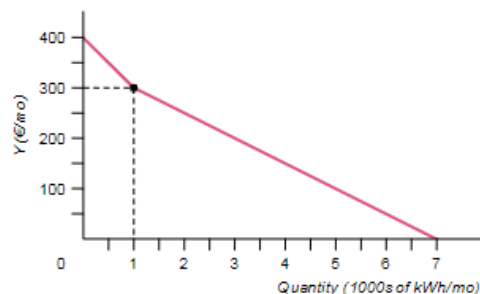
If we want to present the choice between more than 2 goods, we use Marshall's **composite good**. Convert all goods to the system of one specific good X and one composite good Y. A composite good is the set of goods you can buy after spending on good X and is conventionally priced at €1/unit.

The conventional good is also referred to as the numeraire.

Kinked Budget Constraints

The budget constraint is not always a straight line, and thus, can have kinks. This usually happens when the **price of let's say good X is not given but it depends on the quantity bought**.

For example, if the price of Good X is higher when you purchase 1000 units and lower if you buy more than 1000 units, the budget constraint should look like:



Slide 17 ; Chapter 4 [presentation slides]

Ranking Choices: Preferences

Using preference orderings we can rank the preferences of the consumer as follows:

- a. When the consumer **strictly prefers** good A to good B we use the connotation:
Good A \succ Good B
- b. When the consumer is **indifferent** between the two goods we use:
Good A \sim Good B

Properties of the Preference orderings

1. **Completeness:** All bundles can be ranked. (strong property)
2. **More is better:** Larger quantities are more preferred than smaller. This allows us to identify which bundles make consumers indifferent. In reality, though, there are cases in which more-is-worse or less-is-better. (weak property)
3. **Transitivity:** If you prefer A to B, B to C, then you prefer A to C. This property prevents cycling problems.
4. **Convexity:** Averages and mixtures are more preferred to extremes. (related to the decreasing marginal return)

Utility function

It is used to assign a numerical value to each possible consumption bundle and rank all the bundles in the budget set.

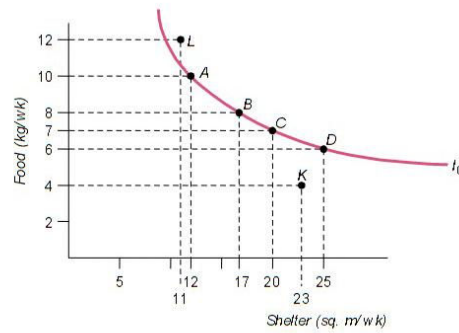
$$A \succ B \Leftrightarrow U(A) \geq U(B)$$

The utility function must satisfy the following conditions:

1. Should be **increasing** in both x and y: $f'_x(x,y), f'_y(x,y) \geq 0$
2. Should be **concave**: $f''_x(x,y), f''_y(x,y) \leq 0$
3. **Separable** in x and y: $U(x,y) = u(x) + v(y)$

Indifference Curves

Indifference curves represent the set of bundles that provide the consumer with the **same level of utility**, i.e. all points on the curve provide them with the same level of satisfaction.



Slide 36; Chapter 4 [presentation slides]

Properties of the Indifference curve

1. There **exists only one indifference curve** for each value of \bar{U}
This property relates to the property of completeness.
2. Indifference curves **cannot cross**.
This property relates to the property of transitivity and more is better.
3. Indifference curves are **downward sloping**
This property relates to the more is better property.
4. All points lying **above an indifference curve are preferred to those lying on it**.
This relates to the 'more-is-better' property.
5. Indifference curves are **less steep when we move downward to the right**.
This relates to the 'more-is-better' property.
6. Indifference curves **further away from the origin have higher utility**.
This relates to the 'more-is-better' property.

Note: People with different tastes have different indifferent maps.

Interpreting indifference curves

Indifference curves that are steeper indicate that a consumer prefers the good on the X axis. If the indifference curve lies flat, the consumer prefers the good on the Y axis.

Computing indifference curves

An indifference curve is defined as $(x,y): U(x, y) = \bar{u}$.

This shows the set of points x and y such that the utility the customer gets from x and y is equal to the level \bar{u} of utility.

For example if an indifference curve is defined as: $U(x, y) : cx + dy$, where $\bar{U} = 7$

The indifference curve will thus be:

$$\begin{aligned}7 &= cx + dy \\ y &= 7/d - c/d x\end{aligned}$$

Marginal Rate of Substitution

MRS tells us how many **extra units of good X** is needed in **exchange for a reduction of good Y to get the same utility** U . It is also the slope of the indifference curve and is calculated using partial derivatives of U according to x and according to y . Take note that the slope is not always constant along the indifference curve. MRS gets smaller as we move to the right (decreasing marginal return). It is the total differential of the indifference curve.

$$MRS(x, y) = - U_x / U_y$$

Indifference curves are normally downward sloping, that is why there is a negative.

Separable Utility Function

These functions are not linear functions of x and y . $U(x, y)$ is separable in the arguments:

$$U(x, y) = g(x) + h(y)$$

With g and h concave and increasing.

$$\text{For example, } U(x, y) = \ln x + \ln y$$

$$\Rightarrow U_x = 1/x \text{ and } U_y = 1/y$$

$$\text{Thus, } MRS = -y/x$$

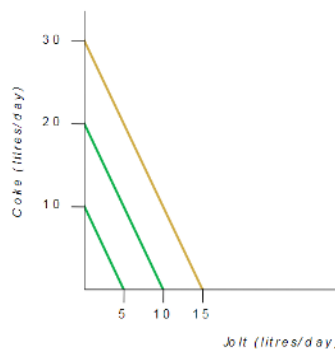
Perfect Substitutes

Two different goods that **provide the same utility** are said to be substitute goods. Eg. roses and tulips.

The slope, indicating the substitutability of x and y , is constant and **equals** $-a/b$. The indifference curve is a **downward-sloping straight line**. In case of perfect substitutes, the utility function and MRS are given by:

$$U(x, y) = ax + by$$

$$MRS = -a/b$$



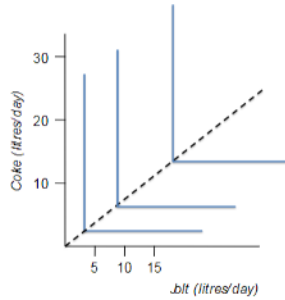
Slide 46; Chapter 4 [presentation slides]

Perfect Complements

Two goods that are always used in a **certain ratio** or in pairs are considered to be complementary goods. The utility function is

$$U(X, Y) = \min \{cx, dy\}$$

Customer's indifference curve is **L-shaped with** $MRS = \infty$ on the vertical and $MRS = 0$ on the horizontal side, making the increase of only one good not effective in raising utility. At the corner where $cx = dy$, the derivative does not exist. The corners of indifference curves form a straight line with slope c/d .



Slide 48; Chapter 4 [presentation slides]

Irrelevant Goods

In cases when the customer is not interested in one of the two goods at all, his/her indifference curves are **straight lines**.

Bads

Of the two goods, there is one good X that the customer wants to reduce as much as possible, called "bad". The indifference curves are **increasing but convex**. The fundamental properties must still hold.

Combining Budget Constraints and Preferences

To maximise utility within your affordability, we need to find the **optimal bundle**. This is a point that is on the budget line as well as on the highest possible indifference curve.

We can have two possible solutions:

1. **interior solution** (consume a strictly positive amount of all goods)
2. **corner solution** (only consume one of the goods)

For the interior solution, we choose the point of tangency between the budget line and the highest indifference curve.

$$\text{This is where: } MRS = -U'_X/U'_Y = -P_X/P_Y$$

So, the problem we are facing is the maximisation of $U(X,Y)$ subject to $P_X X + P_Y Y = M$. The method we use varies between the two possible solutions.

Microeconomics – IBEB – Lecture 4, week 2

Steps to get the interior solutions

To find the **interior solutions** we use the Lagrangian function.

1. Write the \mathcal{L} **function and its first-order conditions**.
2. Take the **ratio of \mathcal{L}'_X and \mathcal{L}'_Y to get rid of λ**
3. **Rewrite one variable with respect to the other**
4. **Plug it into the budget constraint** and solve the new equation
5. **Plug the new-found value back into either the initial budget constraint** or the function (whichever is easier) to find the remaining value.
Afterwards, check if: both X and Y are positive and $P_xX + P_yY = M$.

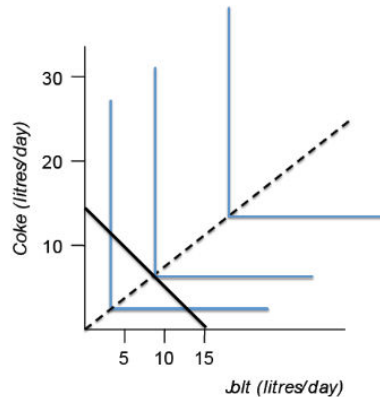
Alternative Method: Direct substitution of budget constraint

1. From the budget constraint, **rewrite x with respect to y** and plug it into the utility function to get an **unconstrained maximisation problem** with one variable.
2. Then, **find the derivative of the function**, equalise it to 0, solve for y.
3. Plug it back into the **initial budget constraint to solve for x**.

In special cases of perfect complements, **the derivative at the corner point** cannot be determined. In such a situation, we solve a system of two equations:

1. The **line which is defined by connecting the corners** of the curves.
2. The **budget constraint**.

The solution is the optimal choice.



Slide 72; Chapter 4 [presentation slides]

Steps to get the corner solutions

To get the corner solutions, there are no tangency points, so the optimal bundle is at either end of the budget constraint.

1. **MRS < PX/PY (flatter** indifference curve than budget line) $\rightarrow x = 0, y = \max$
2. **MRS > PX/PY (steeper** indifference curve than budget line) $\rightarrow y = 0, x = \max$

More on utility

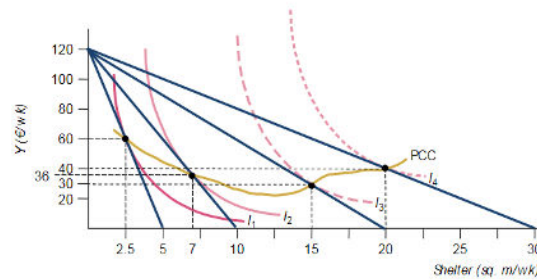
1. **Ordinal function:** the utility function only tells the ranking, not the intensity of the preferences
2. **Affine transformations:** turn $u(x)$ into $v(x) = a + bw(u(x))$ to calculate derivatives easier, where a is a real number, $b > 0$ and w is increasing.
3. **Quasi linear function:** $U(x, y) = y + v(x)$ In this function, X is the necessity/public good and Y is the composite good. The demand of X is independent of one's income.

Individual Demand and other curves

In order to derive individual demand curves, we need to use the rational choice model and introduce concepts such as **income effect, substitution effect and elasticity**.

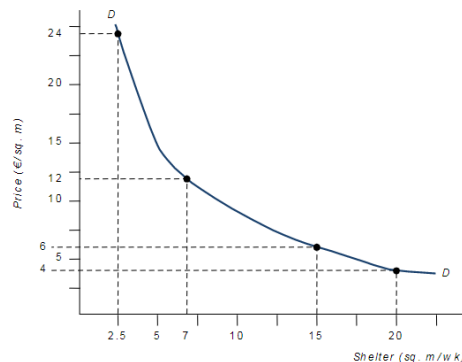
In order to focus on a specific good X and its demand, we use the composite good representation.

Let P_X vary while keeping income M and P_Y constant to see the different optimal bundles. The line connecting those bundles is the Price consumption curve (PCC). Through the PCC, we see how the demand for good X changes as its price changes.



Slide 3; Chapter 5 [presentation slides]

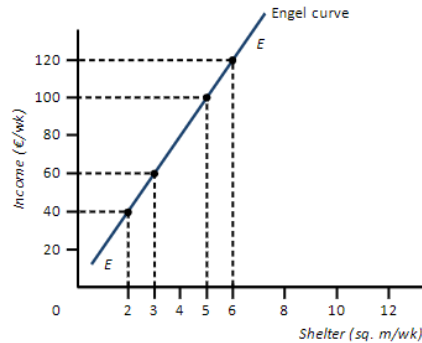
The same information provided by the PCC curve can also be shown as a function of P_X in the (x, P_X) space. Plotting the series of optimal x^* in this way will derive an individual demand curve, which is downward sloping.



Slide 4; Chapter 5 [presentation slides]

The income-consumption curve (ICC)

Let income M vary while keeping the prices constant and see how demand for good X changes. The ICC shows how changes in income affect consumer consumption/demand. By plotting the relationship between income and the demand for x in the (x, M) domain, we get the Engel curve. This curve is not always a straight line.



Slide 5; Chapter 5 [presentation slides]

Types of Goods

In order to split goods into different types, we need to see how the increase in income impacts the demand for the good.

1. Normals Goods

Goods whose demand grows with income,

- a. **Luxury Goods:** Demand grows faster than income.
- b. **Necessity Goods:** Demand grows slower than income

2. Inferior Goods

Goods whose demand shrinks with income.

This is because these goods are easily substituted with more expensive and preferred goods as we become richer.

Effect of Price Change

The effect of the price change can be split into two effects: substitution effect and income effect.

Thus, **Total effect: income effect + substitution effect**

Substitution effect

Changes in demand for good X resulting only from the **change in relative attractiveness** of good X compared to other goods (in terms of price), keeping utility constant. It has the opposite direction to that of the price change and thus is always negative. More specifically, when the price of good X goes down, the demand for X goes up and vice versa.

Income effect

Results from the change in customers' real purchasing power for a good X. It is negative for normal goods but positive for inferior goods. So, for inferior goods, if the price decreases, the income effect can be a decrease in demand. More specifically, when someone is richer, they will buy more normal goods and fewer inferior goods, and vice versa.

In the case of Giffen Goods is when the income effect is positive and dominates the substitution effect.

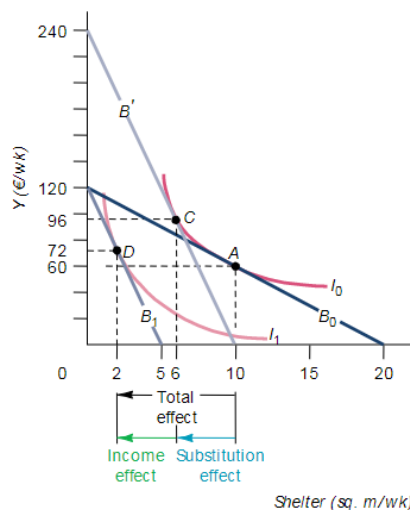
Calculating the substitution effect and income effect

Solving graphically

Substitution effect is found at the point of tangency of original indifference curve and the line whose slope correlates to the new relative prices.

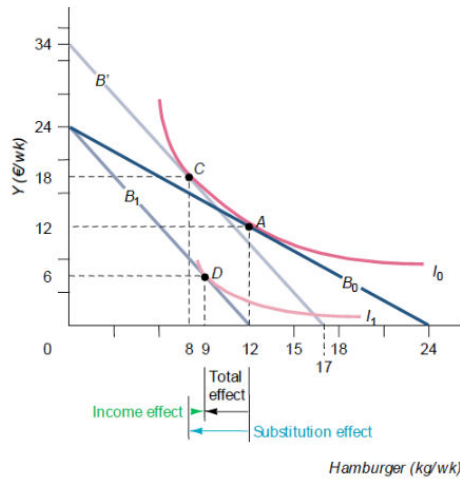
Income effect is the difference between the point of the substitution effect, and the point of the new optimal bundle.

Normal Goods



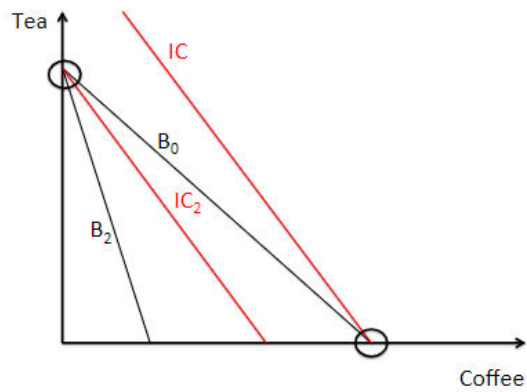
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Inferior Goods



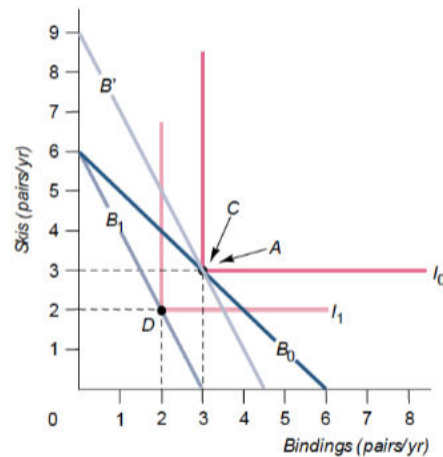
Slide 14; Chapter 5 [presentation slides]

Perfect substitutes (Full substitution effect and no income effect)



Slide 17; Chapter 5 [presentation slides]

Perfect complements (no substitution effect, only income effect)



Slide 15; Chapter 5 [presentation slides]

Solving algebraically

Substitution effect

For a price change on good x from P_x to P'_x proceed as follows: -

1. solve the consumer's problem for the initial price P_x finding x^* , y^* and U^*
2. solve for the new chosen quantity x^{**} with P'_x when utility remains at the same level U^*
3. solve the system $MRS = P'_x/P_y$ and $U(x^{**}, y^{**}) = U^*$

Income effect

1. solve the consumer's problem for the new price $P'_x \rightarrow x^\dagger$
2. compute the difference between x^\dagger find x^{**}

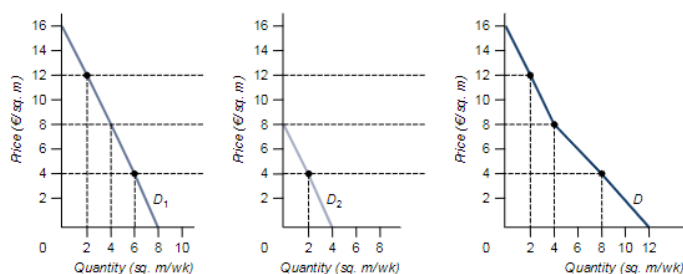
Microeconomics - IBEB - Lecture 5, week 2

Deriving Market Demand

The aggregate demand is calculated through the process of horizontal summation. The process is hence about adding the quantities of the goods demanded and not the price. Thus, the demand function is given in the form:

$$q_i = \alpha_i - b_i P$$

Following this, the functions are added together.



Slide 22; Chapter 5 [presentation slides]

Special Case 1

Demand where intercept doesn't depend on the identity of the consumer.

If all agents have the same intercept with the price axis (i.e. demand function is given by $P = a - c_i q_i$), then the x-axis intercept is the summation of the individual quantities.

Special Case 2

If all n agents are identical with demand function $P = a - b q_i$, then the x-axis intercept is equal to n times the individual demand.

Following this we know:

- a. The intercept with the vertical axis of the market demand is the same as that of every individual.
- b. The intercept of the horizontal axis of the market demand is the product of the individual demand times n.

Elasticities

Elasticity can be defined as the measure of the change in demand following a change in an economic variable such as price or income.

Own good price elasticity

The percentage change of demand for good X following 1% change of price of X. This is always negative and ranges between $-\infty$ to 0. It can be calculated by:

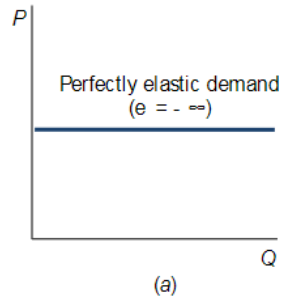
$$\varepsilon = \frac{P}{Q} \frac{dQ(P)}{dP} \simeq \frac{\frac{\Delta Q}{Q}}{\frac{\Delta P}{P}}$$

When:

- a. $|\varepsilon| > 1$; the demand for good X is elastic and a price reduction increases the total expenditure.
- b. $|\varepsilon| < 1$; the demand for good X is inelastic and a price reduction decreases total expenditures.
- c. $|\varepsilon| = 1$; the demand for good X is unit elastic and expenditure is at its maximum.

Perfectly elastic demand

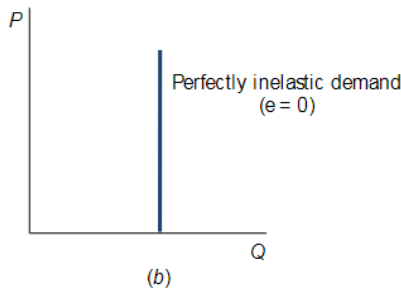
This is when the demand for a good is completely dependent on its price. Thus, at a particular price, consumers would want to buy any quantity of the good; below that price, they would want to buy infinite goods and above that price it would be 0.



Slide 31; Chapter 5 [presentation slides]

Perfectly inelastic demand

Here, people would buy the same amount of goods regardless of the price of the goods.

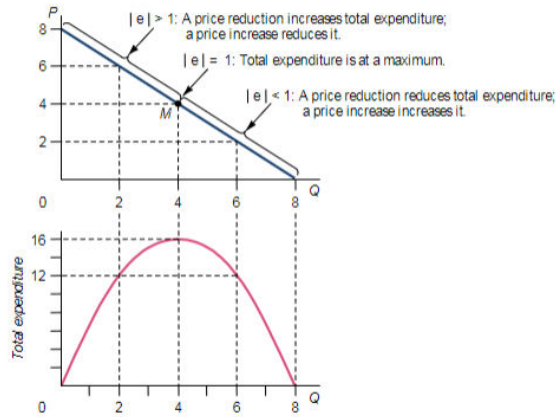


Slide 31; Chapter 5 [presentation slides]

Important: A linear demand has a constant slope but not a constant elasticity.

How much should the producer sell in the market in order to maximise expenditure?

⇒ Revenue curves are bell shaped (parabolas) and thus, the point that should be chosen is when the quantity corresponds to the top of the bell curve; which turns to be the point at which the demand for the good is unit elastic.



Slide 38; Chapter 5 [presentation slides]

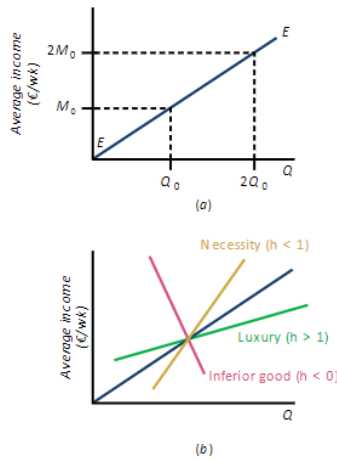
Income elasticity

Analogous to own-good price elasticity:

$$\eta = \frac{M}{Q} \frac{dQ(M)}{dM} \simeq \frac{M}{Q} \frac{\Delta Q}{\Delta M}$$

For normal goods: $\eta > 0$; Luxury goods: $\eta > 1$; Necessities: $0 < \eta < 1$

For inferior goods: $\eta < 0$



Slide 40; Chapter 5 [presentation slides]

Cross Price elasticity

The computation of the elasticity of demand for good X as a function of changes in the price of good Z is:

$$\epsilon_{xz} = \frac{P_z}{Q} \frac{dQ_x}{dP_z} \simeq \frac{P_z}{Q} \frac{\Delta Q_x}{\Delta P_z}$$

For substitutes the elasticity is positive, whereas for complements it's negative.

Other ways to compute elasticity

1. Segment ratio approach: this is a geometric approach to find elasticity.
2. Arc elasticity: used when you only know 2 points on the demand curve.

Applications of the Rational choice model

The rational choice model previously discussed focused on the optimal choice between different goods. The first application of this model considers choice related to time. Here, we care about saving and borrowing.

Intertemporal Choice Model

Instead of choosing between two goods, this model chooses between consumption today (C1) and consumption tomorrow (C2).

Let M1 be today's income and M2 be tomorrow's income (in tomorrow's currency). Suppose there is no price inflation and price of C2 equals 1. As in the basic rational consumer choice model, we put C1 on the x-axis, C2 on the y-axis and solve the problem. In order to solve this problem, we apply the tools from chapter 4.

Intertemporal Opportunity cost

In order to compare consumption today with consumption tomorrow, we need to express them in the same unit of measurement. To do this, we need to look at the intertemporal opportunity cost.

The net opportunity cost of consumption today with respect to consumption tomorrow is the interest one could earn by leaving the money in the bank. The gross opportunity cost of one unit of today's consumption is $(1 + r)$. To transform today's price to tomorrow's price we multiply it by $(1 + r)$, while tomorrow's price to today's price requires us to divide it by $(1 + r)$.

Intertemporal Budget Constraint

If the consumer spends all his income today, he can consume:

$$C_1 = M_1 + \frac{M_2}{1+r}$$

The slope of the budget constraint is $-(1+r)$.

If one spends all their income tomorrow, they can consume:

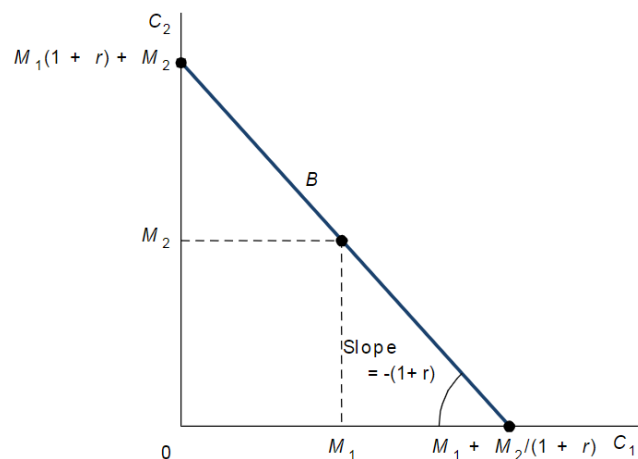
$$C_2 = M_2 + M_1(1+r)$$

Thus, the intertemporal budget constraint in today's currency:

$$C_1 + \frac{C_2}{1+r} \leq M_1 + \frac{M_2}{1+r}$$

The intertemporal budget constraint in tomorrow's currency:

$$(1+r)C_1 + C_2 \leq (1+r)M_1 + M_2$$



Slide 10; Chapter 6 [presentation slides]

Note: when the rate of borrowing is higher than the rate of lending, there is a kink at the endowment point of the budget constraint.

Microeconomics – IBEB – Lecture 6, week 2

Intertemporal indifference curves

In chapter 4, we used MRS to determine the slope of an indifference curve. Here, we use **Marginal Rate of Time Preference (MRTP)**.

$$\text{MRTP} = - U'_{C1} / U'_{C2}$$

This Marginal Rate of Time Preference (MRTP) is basically the MRS but for the intertemporal consumption model. The problem the consumer faces thus becomes:

$$\max_{C_1, C_2} U(C_1, C_2) - \lambda \left(C_1 + \frac{C_2}{1+r} - M_1 - \frac{M_2}{1+r} \right)$$

Optimal intertemporal consumption bundle

The optimal condition is where the slopes, indifference curve and the budget constraint are equal. In the intertemporal model, this condition is known as the Euler equation.

$$\text{MRTP}(C1^*, C2^*) = - (1+r)$$

Saving and Patience

If $C1^* < M1$, the consumer is not spending all his present income and is saving:

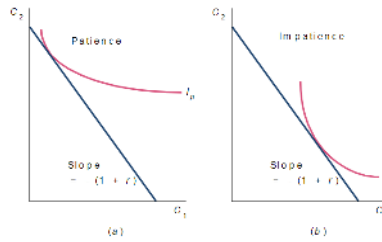
$$S1 = M1 - C1$$

Tomorrow he can consume:

$$C2 = M2 + (1+r) S1$$

$\text{MRTP} < 1 + r$: the consumer is **patient**

$\text{MRTP} > 1 + r$: the consumer is **impatient**



Slide 18; Chapter 6 [presentation slides]

If the indifference curve (IC) pointed down is steeper than the budget constraint, the consumer is considered as impatient. Therefore, consumer is impatient at (M_1, M_2) :

$$MRTP > 1 + r$$

If the indifference curve (IC) pointed down is flatter than the budget constraint, the consumer is considered as patient. Therefore, consumer is patient at (M_1, M_2) :

$$MRTP < 1 + r$$

Modelling patience using utility function

We can model patience by introducing parameter into the utility function:

$$u(C_1, C_2) = u(C_1) + \beta u(C_2)$$

where $u(C)$ is any increasing and concave function. r is the gross rate of time preference, δ is the net rate of time preference. The formula for :

$$\beta = \frac{1}{1 + \delta}$$

The Euler equation becomes:

$$\frac{U_{C_1}}{\beta U_{C_2}} = 1 + r \Leftrightarrow \frac{U_{C_1}}{U_{C_2}} = \beta(1 + r) = \frac{1 + r}{1 + \delta} \geq 1$$

The following cases might occur:

Case 1:

$$\frac{U_{C_1}}{U_{C_2}} = \frac{1 + r}{1 + \delta} = 1 \stackrel{r=\delta}{\Leftrightarrow} U_{C_1} = U_{C_2} \Leftrightarrow C_1^* = C_2^*$$

Case 2:

$$\frac{U_{C_1}}{U_{C_2}} = \frac{1 + r}{1 + \delta} > 1 \stackrel{r > \delta}{\Leftrightarrow} U_{C_1} > U_{C_2} \Leftrightarrow C_1^* < C_2^*$$

Case 3:

$$\frac{U_{C_1}}{U_{C_2}} = \frac{1 + r}{1 + \delta} < 1 \stackrel{r < \delta}{\Leftrightarrow} U_{C_1} < U_{C_2} \Leftrightarrow C_1^* > C_2^*$$

Effects of changes in r , M_1 , M_2

Change in r

If r increases, the opportunity cost of present consumption increases.

If r decreases, the opportunity cost of present consumption decreases.

Given an increase of r , the substitution and income effect of C_1 is as follows:

| | Borrower | Lender |
|---------------------|----------|--------|
| Substitution effect | - | - |
| Income effect | - | + |
| Total effect | - | -/+ |

The signs refer to the change in C_1 as $1 + r = P_{C_1}$

Changes in M_1 and/or M_2

Any change in income impacts both C_1 and C_2 . Here we have 3 possible situations.

1. **Change in current income**

Reflected less than one-for-one in the change in current (and future) consumption.

2. **Change in future income**

Reflected less than one-for-one in the change in future (and current) consumption.

3. **Change in permanent income** (both present and future)

Reflected by (nearly) identical change in both present and future consumption.

These cases are argued through **Friedman's permanent income hypothesis**.

Another way to explain them is:

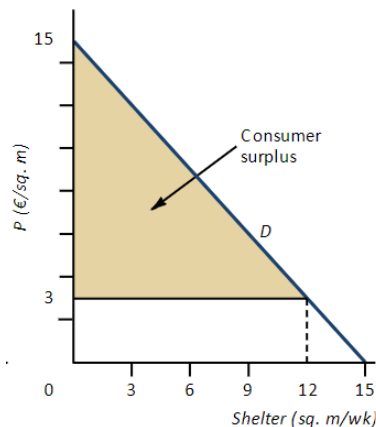
- 1) We consume permanent changes in income, BUT
- 2) Save a large proportion of temporary changes in income.

Consumer and Producer Surplus

Willingness to pay vs the price actually paid

All consumers pay the identical price in the market, P^* . However, some consumers value the goods more than the market price. Therefore, these consumers are making a profit.

The difference between how much the consumer is willing to pay and actually pays is the **consumer surplus**. It is given by the area below the demand curve and above horizontal line corresponding to P^* .



Slide 32; Chapter 6 [presentation slides]

Sellers can transfer some of this surplus to them using a two-part pricing strategy.

Two part tariffs

The simplest way of using this strategy is as follows:

1. Charge no rental price for use of the good;
2. Charge a price for access to the good that is at most equal to the consumer's surplus.

Economic theory beats intuition

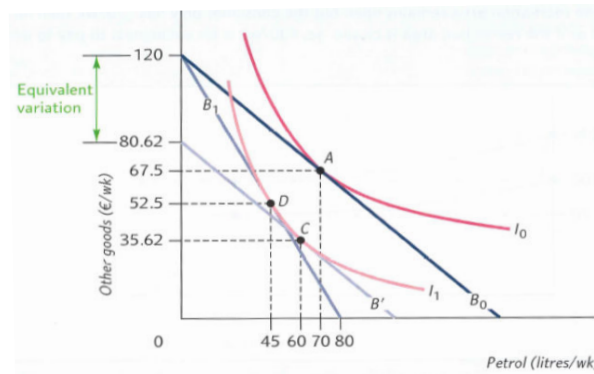
Whenever economic conditions change, reconsider your previous choices using the model and you can still be better off.

Effect of taxes

Equivalent variation

In order to see how much consumers are dissatisfied with taxes, i.e. what is the tax effect on consumer choice, we can use **equivalent variation**. We can find it in three steps:

1. Find consumer's optimal consumption choice BEFORE tax;
2. Find the new optimal consumption choice AFTER tax increase;
3. Find the amount of income the consumer would be willing to give away, in order not to face price increase but have the same level of utility as before.



Slide 48; Chapter 6 [presentation slides]

Commodity taxes and elasticities

The economic share of the tax burden, supported by each side of the market, is a function of elasticities. We can show this using the **own-price elasticities of demand and supply**.

When the demand and the supply curves are *constant elasticity* ones, we have:

$$Q^D(P) : \frac{\Delta Q^D}{Q} \frac{P}{\Delta P^D} = \varepsilon^D \qquad Q^S(P) : \frac{\Delta Q^S}{Q} \frac{P}{\Delta P^S} = \varepsilon^S$$

We can observe that these demands are not lines, but rather hyperbolas.

Demand-over-supply ratio is given by (note: the initial, no tax equilibrium will be labelled as (Q^*, P^*)):

$$\frac{\Delta P^D}{\Delta P^S} = \frac{P^{tD} - P^*}{P^{tS} - P^*}$$

The new equilibrium is such that , which yields the following:

$$\frac{\Delta P^D}{\Delta P^S} = \frac{\varepsilon^S}{\varepsilon^D}$$

Microeconomics - IBEB - Lecture 7, week 3

A crash course in probability theory

Deterministic variable: Variable that takes a certain value for sure (e.g. a two-euro coin is worth 2 euros).

Random variable: Variable that can take any of a number of values and for which there is uncertainty.

Event: Fact of one specific event happening.

Support of the distribution: Set of the values a random variable can take.

Discrete random variable: If the variable can only take a finite number of values (e.g. coin, dice...).

Continuous random variable: If the variable can take all the real values contained in some interval.

Probability

The chance that a random variable X takes on a specific value x among its possible set of values. This is usually written as p or $Pr(X = x)$.

Probability Distribution

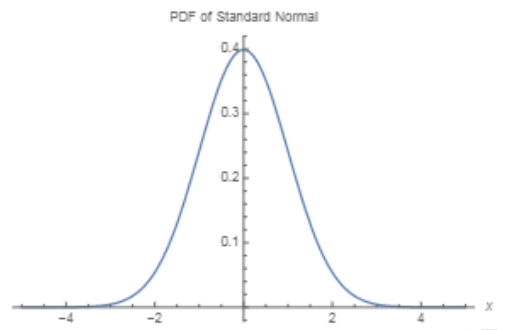
It tells us how the chances of each possible value that a random variable can take are distributed. Probability is always $0 \leq p \leq 1$. The sum of all p must equal to 1. If each

event has the same chances of happening, and there are n possible events, probability of any event is $1/n$. For a continuous random variable, $\Pr(X = x) = 0$.

Probability Density Function (PDF)

Continuous random variable

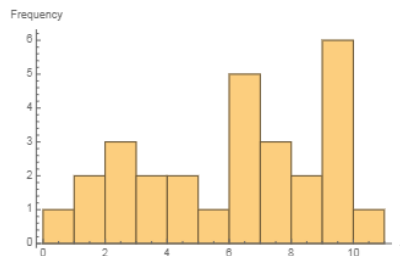
It is a continuous function that tells how relatively likely are the events in the support of a random variable. The PDF of **Standard Normal distribution**:



Slide 7; Crash course in Probability [presentation slides]

Discrete random variable

It looks at the frequency with which the random variable could take on each value. To report this frequency distribution, we use **histograms**:



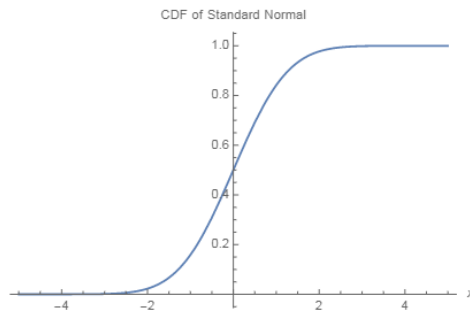
Slide 8; Crash course in Probability [presentation slides]

Cumulative probability Distribution Function (CDF)

CDF is a continuous function increasing from 0 to 1 that tells how likely it is that any realisation takes on a value less than a real number x . Its notation is $\Pr(X < x)$.

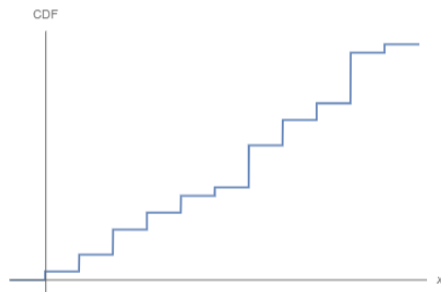
$\Pr(X < -\infty) = 0$ and $\Pr(X < +\infty) = 1$.

Standard normal distribution



Slide 10; Crash course in Probability [presentation slides]

Discrete random variable



Slide 11; Crash course in Probability [presentation slides]

Expected value or Average

Average, mean or expected value of a random variable (E) is the weighted sum of all possible values, with weights equal to the probability of each value happening.

Discrete random variable:

$$E(x) = \sum_{i=1}^N p(x_i) * x_i$$

Where $P(x_i)$ is the probability of the data point to appear, and x_i is the data point.

Example:

$$E(\text{dice}) = \frac{1}{6} (1+2+3+4+5+6) = 3.5$$

Continuous random variable:

$$E(x) = \int_{\text{sup}(X)} x * f(x) dx$$

Example:

$$E(\text{lottery}) = \frac{1}{99} \sum_{i=1}^{99} i = 50$$

The uniform probability distribution

In a uniform distribution, every event has an equal probability to occur.

A continuous probability distribution whose support is some finite interval $[a, b]$. Any value in $[a, b]$ has the same probability of happening. Its mean is equal to:

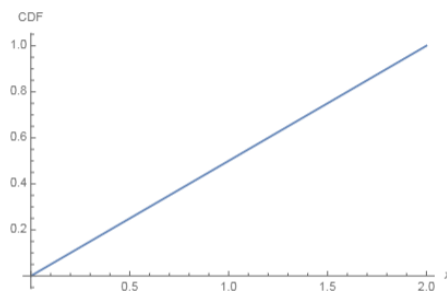
The CDF is: $F(x) =$

$$\frac{x-a}{b-a}$$

The PDF is constant: $f(x) =$

$$\frac{dF(x)}{dx} = \frac{1}{b-a}$$

The uniform distribution over $[0,2]$ is:



Slide 16; Crash course in Probability [presentation slides]

And the density (PDF) is the slope of the above line, which is a constant line at $1/2$.

After drawing a value x from the support of the distribution, the expected value of the next draw is:

if lower than x it is:

$$\frac{x+a}{2}$$

if higher than x it is:

$$\frac{b+x}{2}$$

Probabilities in Economics

Investment

Suppose three possible values at the end of the year of a €100 stock investment are: 0, 200, 300 and their respective probability distribution is $1/5, 1/2, 3/10$.

The **expected value of the investment** is:

$$E(I) = 0 * 1/5 + 200 * 1/2 + 300 * 3/10 = 190$$

The **expected value of the gain on investment** is:

$$E(\text{gain on } I) = -100 * 1/5 + 100 * 1/2 + 200 * 3/10 = 90$$

Choice under uncertainty and Economics of information

The rational choice model assumes **perfect information**. In practice, however, most decisions are made **under uncertainty**.

Economics of information

Besides parties who have similar goals, there are those with misaligned preferences because their objectives do not coincide. E.g. an insurance company and a driver. In these cases, **information** plays a very important role. Also, **communication** (not only verbal) can serve as a useful **signalling device**.

Signalling

Signalling is communication that conveys information. This type of communication need not be verbal. Signals can be interpreted as actions. Thus, signalling is communication (through actions) that conveys information about oneself. It has two properties:

1) Costly-to-fake principle

For a signal to be credible, it must be difficult, or costly, to fake. Otherwise, it does not give much information and will be ignored. E.g. having exceptional acrobatic skills, driving a Ferrari, etc.

2) Full disclosure principle

Individuals are forced to reveal even unfavourable information about themselves as a result of the disclosure of others' favourable information. Otherwise, their silence will be considered as a signal of much inferior values. E.g. when a company offers 5-year guarantees for their product, other companies also have to offer it. Otherwise, customers will think that their products are terrible.

Applications of signalling

Application 1: finding a partner

Use costly-to-fake signals to gather information about potential partners.

Application 2: professional guilds

View professions based on their costly-to-fake barriers to entry.

Application 3: Akerlof's market for 'lemons'

Owners value good cars more than bad ones. As used-car buyers notice that a large portion is bad, the price of used cars falls. Owners of high-quality cars lose their incentive to sell them. Thus, only bad cars enter the used-car market. This is an example of **market failure** due to **asymmetric information**.

Choice under uncertainty

In an uncertain world, a decision made is similar to a gamble. We assume utility comes from the monetary return on one's choices, denoted by $U(M)$ where M is money – a deterministic value – and $U(x)$ where x is a possible realisation from the lottery – a random variable.

The utility function, called **Von Neumann Morgenstern (VNM) Expected Utility function**, assigns a utility to each possible realisation and holds all properties of the rational choice model except for decreasing marginal return. So, the function $U(x)$ is always defined and increasing.

The decreasing marginal return (one possible case) is linked to the consumers attitude towards risk.

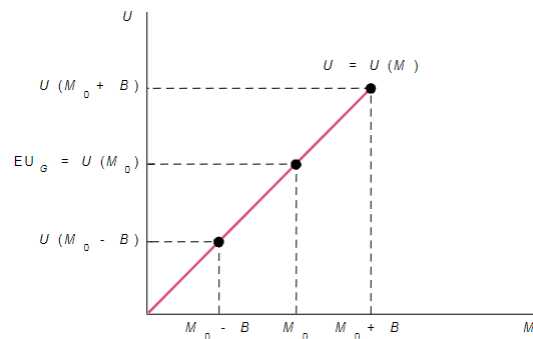
Expected utility of a gamble $E(U)$ is the expected values of utility from all possible outcomes. Agents can have 3 attitudes toward risks: aversion, neutrality and risk-seeking.

Attitude towards risk

The VNM function satisfies completeness and the more is better the axiom. That means that $U(x)$ is always defined and increasing. The second order derivative of this utility function tells us whether an agent is risk-averse, risk-neutral, or a risk-lover.

Risk neutrality

The utility function is an upward-sloping straight line such that $U'' = 0$.

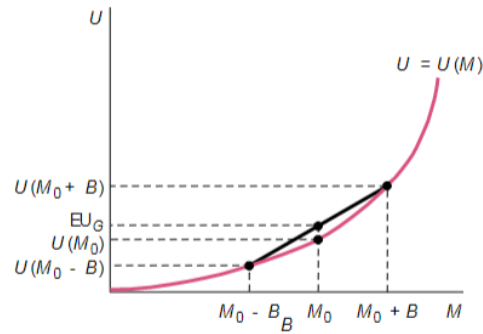


Slide 23; Chapter 7 Part 1 [presentation slides]

A risk-neutral agent regards money or utility as one and the same thing to some extent. Therefore, they are indifferent between a fair gamble with mean M and getting M for sure. The linear utility function does not have marginal utility.

Risk seeker

The utility function is increasing but convex:

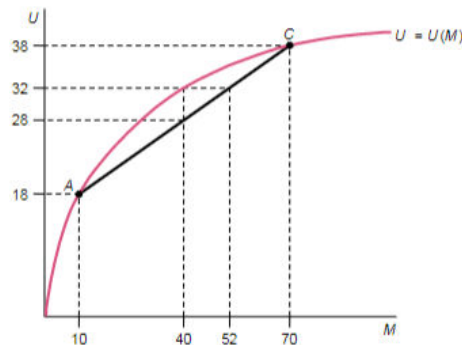


Slide 25; Chapter 7 Part 1 [presentation slides]

A risk-lover accepts the gamble because, when upside and downside have the same probability and absolute value, they value the upside of a gamble more than its downside. However, the fact that a risk-lover prefers a gamble does not imply that they do not care about risk. This utility says that the consumer has an increasing marginal utility of consumption.

Risk aversion

Generally, the graph of utility function is increasing and strictly concave:



Slide 20; Chapter 7 Part 1 [presentation slides]

A risk-averse agent will always refuse a fair gamble, because the utility from taking it, described by the straight line, is always lower than the utility from not doing so, described by the arc of the utility function above the line.

Microeconomics – IBEB – Lecture 8, week 3

Uncertainty and Risk Averse agents

Certainty Equivalent

This is the income level of the sure action that makes the agent exactly indifferent (in terms of utility) between the gamble and the sure action. It is C such that:

$$U(C) = EU(G)$$

where G is the expected value from the gamble. Because a risk-averse agent is willing to pay to avoid risky actions, the income from the sure action C is lower than the expected value from the risky one of G . The difference between C and G measures how much they dislike risks.

Paying advisors and insurance

As agents are risk-averse, they would like to reduce the negative effects of risk. Thus, they are willing to pay advisors and insurers for information.

Advisors

Assume the agent's decision is irreversible, the advisor is always right and will reveal information in return for a payment. Without an advisor's information about whether a risk-averse agent can win the gamble or not, he/she will not take the gamble and will choose the sure action.

When there is advice, however, that agent must be willing to pay a maximum amount P such that the expected utilities from having an advisor and not having an advisor are equal.

$$EU(G - P) = U(C)$$

Insurers

A risk-averse agent will buy insurance to insure their initial wealth M against losses. The maximum amount he/she is willing to pay for it is P such that the expected utilities from having an insurance and not having an insurance are equal:

$$EU(M - P) = EU(M)$$

Topics that are related to this, but are covered in more detail in other courses, are **risk pooling, risk sharing, moral hazard and adverse selection.**

1) Risk pooling

Pooling together different sources of risk to reduce overall risk.

2) Risk sharing

Sharing different risks among different agents to reduce risk at the group level.

3) Moral hazard

Problems of hidden action which can hurt oneself and other parties involved.

4) Adverse selection

Problems with hidden information that can lead to choosing the wrong option.

Search Theory

Basic job search model

You are faced with some job options and you want to choose the one with the highest pay. The only way to know each one's wage, however, is by paying a €5 cost to examine vacancy.

Suppose:

- Your utility function is: $U(w) = a \cdot w$ where $a > 0$ (you are thus risk-neutral);
- Wages are uniformly distributed random variables on $[100, 200]$;
- You can go back to a job you searched previously with no cost;
- The first vacancy you examine pays €150.

Should you continue searching?

You should continue searching if and only if:

MC of searching once more < MB of searching once more

MB of searching once more is the expected value of doing so, which is:

$$\begin{aligned} & \Pr(\text{offer with higher wage}) * E(\text{utility gain from higher wage}) \\ &= [1 - \Pr(\text{offer with lower wage})] * E[a * (\text{expected value of higher wage} - \\ & \quad \text{current wage})] \end{aligned}$$

$$(1 - F(150)) \cdot \left[a \left(\frac{150 + 200}{2} - 150 \right) \right]$$

$$\left(1 - \frac{150 - 100}{100} \right) 25a = 12.5a$$

MC of searching once more is €5; thus,

$$12.5a > 5 \Rightarrow a > 0.4$$

How large should the wage be for you to accept the offer?

You should accept wage w^* if and only if the expected utility gain from searching again is less than or equal to the cost of doing so:

$$\underline{E(\text{utility gain from searching again})} \leq \underline{\text{MC of searching again}}$$

$E(\text{utility gain from searching again})$ is:

$$EU(G(w^*)) = a * EG(w^*) = \text{Pr}(\text{getting a better offer}) * E(a * \text{wage gain})$$

$$= [1 - F(w^*)] a \left(\frac{200 + w^*}{2} - w^* \right)$$

$$= \left(1 - \frac{w^* - 100}{100} \right) a \left(\frac{200 - w^*}{2} \right)$$

$$= \left(\frac{200 - w^*}{100} \right) a \left(\frac{200 - w^*}{2} \right)$$

$$= a \frac{(200 - w^*)^2}{200}$$

MC of searching again is €5, thus, you should stop searching at:

$$5 \geq a \frac{(200 - w^*)^2}{200}$$

Note:

1. While solving the quadratic equation $E(\text{utility gain}) = \text{MC}$, choose the roots that satisfy the restrictions of the model.
2. MC can be a function instead of a constant.

Statistical representation

Group discrimination

Every individual belongs to a group with an **average characteristic** (e.g., productivity) different from other groups. Then, that individual is taken out of their group for a productivity test.

Assume that the test is half correct, and half not. His/her final productivity will be calculated as:

$$\frac{\text{personal score}}{2} + \frac{\text{group average}}{2}$$

Conclusions:

1. When tests are not very accurate, a worker's productivity is estimated to be somewhere between the result of the test and the population's average.

When two workers score the same, productivity of the one with the higher group's average is higher than the other. Average productivity differences between groups cause **discrimination**, thus they will be paid differently.

Microeconomics - IBEB - Lecture 9, week 3

Production

Production presents any activity that creates utility, both in present and in the future. In this course, we talk about production possibilities available to us for a given state of technology and resource endowments.

The production function

The production function describes how **inputs** (e.g. capital, labour, ideas, time...) are transformed into **outputs**. For example:

$$Q = F(K, L) = K^\alpha L^\beta$$

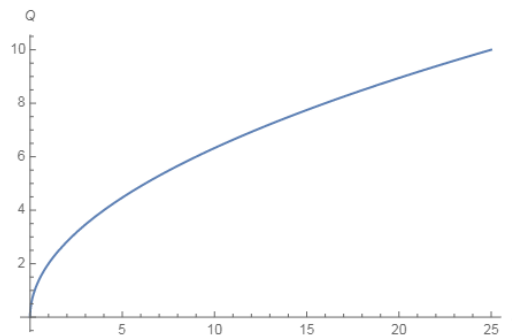
Final goods – goods that do not require any more processing.
 Immediate goods – goods used to produce a final finished product.

Inputs can be fixed or variable. Based on this, we can distinguish between:

- 1) *Short run* – there is at least one input which is fixed.
- 2) *Long run* – there are only variable inputs, whose quantity we can vary.
 - a. Most fixed short-run inputs can become variable in the long run.

Example for short run when capital is fixed:

$$F(K, L) = \sqrt{K}\sqrt{L} = \sqrt{4}\sqrt{L} = 2\sqrt{L}$$

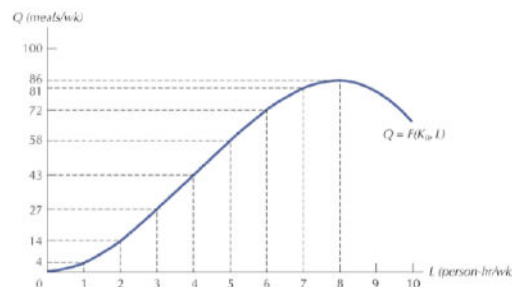


Slide 4; Chapter 10 and 11 [presentation slides]

Diminishing Returns

If a variable input is added with equal amounts and all other inputs are held constant, the resulting increments to output will eventually diminish. This is a short run property.

The assumptions made are that for low levels of production, returns are increasing or constant, but from a point onwards, decreasing marginal return kicks in.



Slide 4; Chapter 10 and 11 [presentation slides]

Negative returns eventually happen because capital stays fixed in the short run, so continuously adding labour inputs can result in too many labourers trying to work with the limited capital.

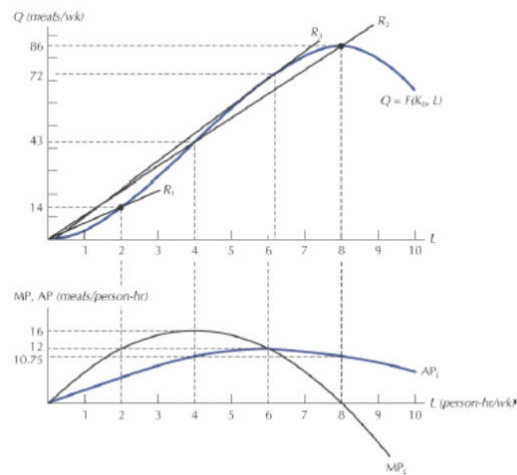
Marginal and Average Product

The marginal product of labour:

$$\frac{\partial F(K, L)}{\partial L}$$

The average product of labour:

$$APL = \frac{F(K, L)}{L}$$



Slide 8; Chapter 10 and 11 [presentation slides]

Based on our assumption on the shape of the production function, MP intersects AP at the maximum of AP. Indeed, the maximum of AP is such that:

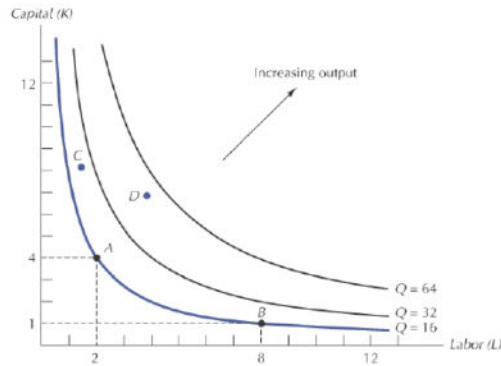
$$\frac{dAP_L}{dL} = 0 \text{ or } \frac{\partial F(K, L)}{\partial L} = \frac{F(K, L)}{L}$$

This essentially means that at this point $MP_L = AP_L$. It is very important to understand the marginal-average distinction here.

The Long Run

Isoquants and MRTS

All inputs are variable. As we vary the inputs, production varies. This is described by **isoquants** - the set of combinations of inputs that give the same output.



Slide 13; Chapter 10 and 11 [presentation slides]

For a production level Q_0 , an isoquant is the pair (K,L) defined by: $Q_0 = F(K,L)$

The marginal rate of technical substitution (MRTS) is the slope of the tangent to an isoquant in any point:

$$\text{MRTS}|_{Q_0} = \left. \frac{dK}{dL} \right|_{Q_0}$$

How will a firm set its production in order to meet its input cost target C ?

The answer is the solution to the problem: $\max_{K,L} F(K,L)$ s.t. $rK+wL=C$.

Isoquants can be likened to indifference curves. While isoquants tell us the sets of combinations of inputs that yield the same output, indifference curves tell us the sets of bundles that would yield the same utility.

Returns to Scale

Returns to scale refers to the reaction of output to a proportional increase in all inputs.

For any given $c > 1$:

- **Increasing Returns to Scale (IRS)**

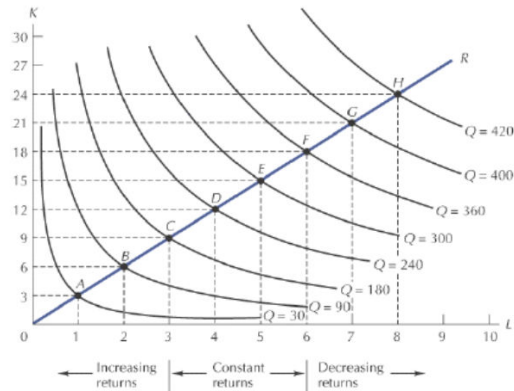
Output increase is *more* than inputs proportional increase: $F(cK,cL) > cF(K,L)$

- **Constant Returns to Scale (CRS)**

Both increases are identical: $F(cK,cL) = cF(K,L)$

- **Decreasing Returns to Scale (DRS)**

Output increase is *lower* than inputs proportional increase: $F(cK,cL) < cF(K,L)$



Slide 17; Chapter 10 and 11 [presentation slides]

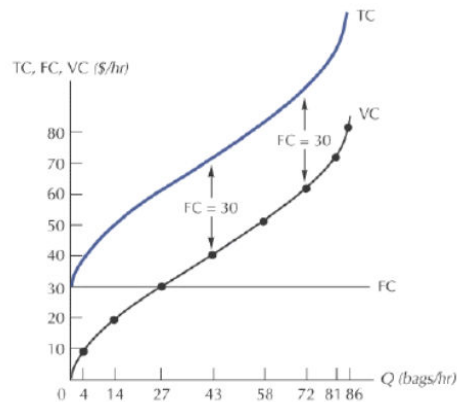
Production Costs: The short run

Fixed costs: Costs of fixed factors of production. Fixed costs incur even without producing anything after all. E.g., the capital in a firm: $FC = rK_0$.

Variable costs: Costs that vary with production. E.g. labour: $VC_{Q_1} = wL_1$ for production level Q_1 .

Total costs: The sum of fixed and variable costs: $TC_{Q_1} = rK_0 + wL_1$

Total costs functions are expressed as a function of the quantity produced Q , thus plotted in the (Q, costs) space.



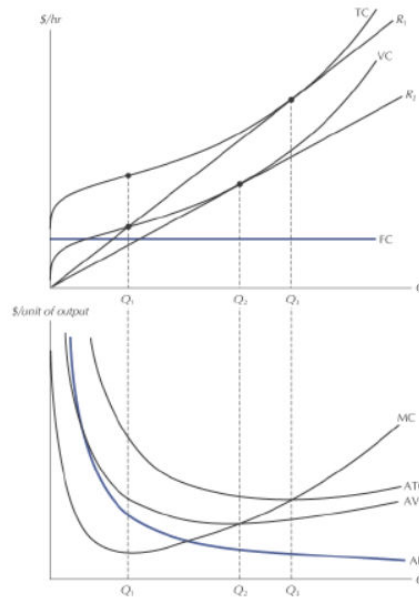
Slide 23; Chapter 10 and 11 [presentation slides]

Computing Cost Functions

Suppose production function is: $F(K,L) = K^{1/2}L^{1/2}$

If $K = 4$, then $rK = 4r$ and $Q = 2L^{1/2}$

$L(Q) = Q^2/4$, Thus, the *variable cost* is: $wL(Q) = w \cdot L$. The total cost is: $rK + wL = 4r + wL$ which is a function of Q .



Slide 26; Chapter 10 and 11 [presentation slides]

Average fixed costs:

$$AFC = \frac{FC}{Q} = \frac{RK_0}{Q}$$

Average total costs:

$$ATC = \frac{RK_0}{Q} + w \frac{LQ}{Q}$$

Average variable costs:

$$AVC = \frac{VC}{Q} = w \frac{LQ}{Q}$$

Marginal costs:

$$MC = \frac{\partial TC}{\partial Q} = \frac{\partial VC}{\partial Q}$$

Minimising wages, Maximising productivity – why?

$$MC = \frac{\partial VC}{\partial Q} ; MP_L = \frac{\partial Q}{\partial L}$$

If labour is the only variable input, then $VC = wL$:

$$MC = \frac{\partial wL}{\partial Q}$$

If wages are fixed, then MC =

$$W \frac{\partial L}{\partial Q} = W \frac{1}{MP}$$

Production costs – The long run

To maximise production, the optimum input mix is such that:

MRTS (K^* , L^*) =

$$\frac{MP_{L^*}}{MP_{K^*}} = \frac{w}{r}$$

In equilibrium, the extra output from the last euro spent on labour must be equal to the extra output from the last euro spent on capital.

Firm's Problem

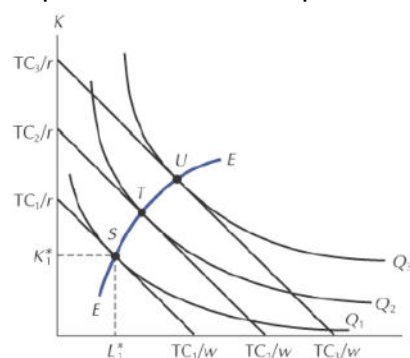
When the firm maximises production subject to a target expenditure on costs C_0 , the optimal level of input the firm should use is the solution to:

$$\max_{K,L} \{F(K,L) - \lambda(C_0 - rK - wL)\}$$

When the firm minimises costs subject to a production target Y_0 , the optimal level of input the firm should use is the solution to:

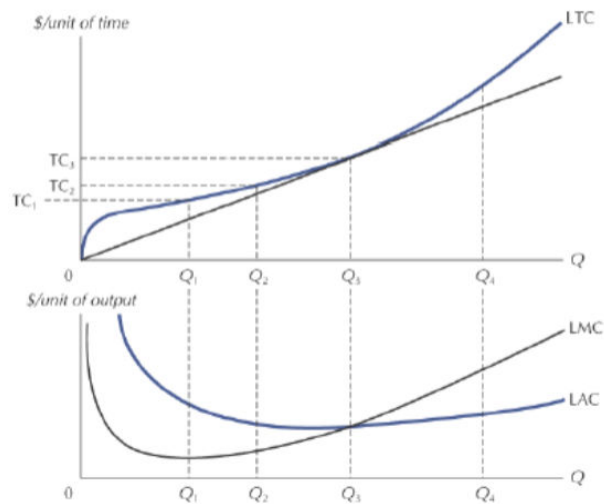
$$\min_{K,L} \{rK + wL - \mu(Y_0 - F(K,L))\}$$

The relationship between the optimal ratios and production is illustrated as follows:



Slide 37; Chapter 10 and 11 [presentation slides]

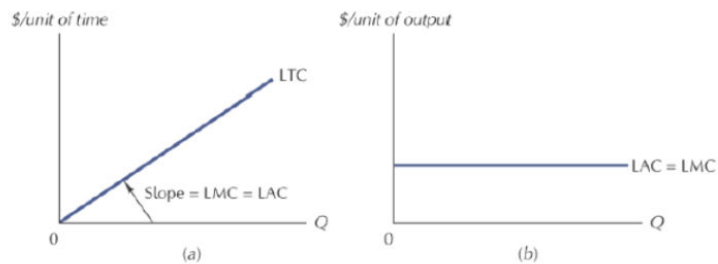
Plotting this relationship in the (Q, TC) space to get the long run total costs curve LTC:



Slide 38; Chapter 10 and 11 [presentation slides]

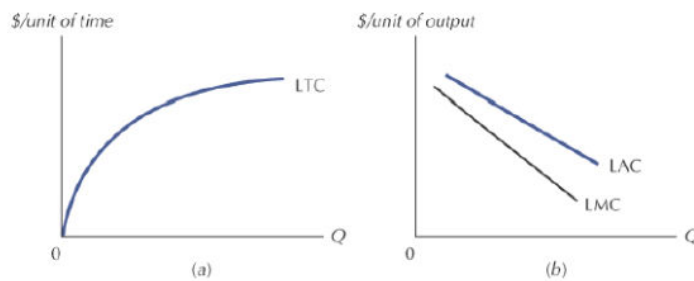
Depending on the type of returns to scale, we can recognize the LTC as:

1) CRS → LTC is a straight **line** through the origin.



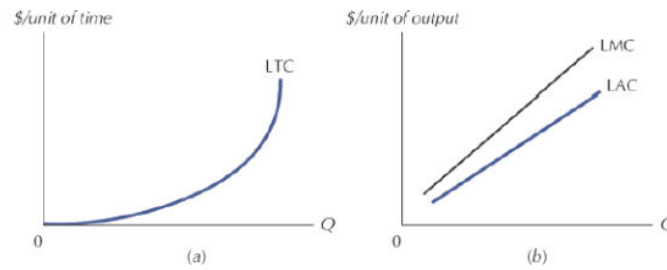
Slide 40; Chapter 10 and 11 [presentation slides]

2) IRS → LTC is **concave** increasing curve from origin (natural monopoly).



Slide 42; Chapter 10 and 11 [presentation slides]

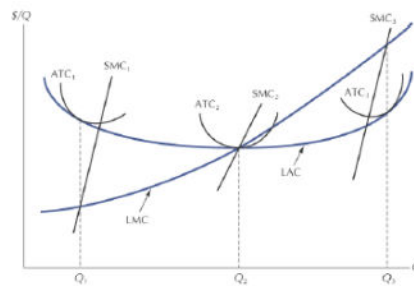
3) DRS → LTC is **convex** increasing curve from origin (market with small firms).



Slide 41; Chapter 10 and 11 [presentation slides]

Wrap up

Let ATC_i denote the short run ATC curve for $K = i$. Then, LAC is the lower envelope of all the ATC_i 's:



Slide 44; Chapter 10 and 11 [presentation slides]

Microeconomics – IBEB – Lecture 10, week 4

In this course, we talk about firms whose owners are also the decision-makers; thus, we avoid the “principal-agent” problem.

Monopoly

A firm that is the single supplier of a product for which no substitutes are available. Also, a monopoly is a firm that has a downward-sloping demand curve. A supply case of monopolists is the simplest due to lack of short-run competition.

Profit Maximising Monopoly

Monopolies want to maximise **economic profits**, which is the difference between total revenue and total costs (both explicit and implicit).

$$\pi = TR - TC = P(Q) * Q - C(Q)$$

*Where $P(Q)$ is the demand function and $C(Q)$ is the cost function.

Maximising profits means ensuring that the difference between MR and MC is 0 (or $MR = MC$).

Total revenue

If the demand curve is $P = a - bQ$, (with a and b holding values greater than 0) Then the total revenue is just the total sales multiplied by the selling price of the quantity sold. Thus, all we need to do is multiply Q to the price function.

$$TR = P(Q) * Q \text{ or } aQ - bQ^2$$

If we ignore costs, the **optimal production point** is at TR curve's global maximum, which can be written down as:

$$\frac{dTR}{dQ} \equiv MR = 0 \Leftrightarrow \mathcal{E} = -1$$

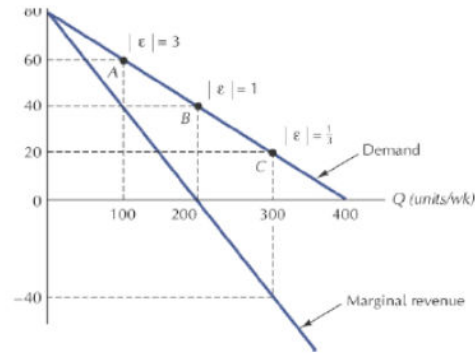
Because,

$$|\mathcal{E}| = \frac{P}{Q} \frac{dQ}{dP}$$

we can prove that,

$$MR = P(Q) * \left(1 - \frac{1}{|\mathcal{E}|}\right)$$

MR line is a downward-sloping line that is twice as steep as demand line and hits 0 when elasticity = -1.



Slide 12; Chapter 13 [presentation slides]

Total costs

$$TC = C(Q)$$

If we ignore revenues, the optimal production point is where costs are minimised:

$$\frac{dC}{dQ} \equiv MC = 0$$

To Remember

1. MC is typically positive for monopolists.
2. Monopolists choose a production level such that MR is equal to MC (thus, the absolute value of the elasticity of demand is greater than 1).
Monopolists never produce on the inelastic part of the demand which allows them to increase profits by reducing sales.
3. Monopolists do not have a supply curve, they just apply the rule $MR = MC$. This optimal condition yields different Q^* and P^* depending on the position of the demand line.

Conclusion

A monopolist will choose the production level such that π is maximised, so:

$$\frac{\partial \pi}{\partial Q} = 0 \Leftrightarrow \frac{\partial TR}{\partial Q} - \frac{\partial TC}{\partial Q} = 0 \Leftrightarrow MR = MC.$$

Therefore, the monopolist:

1. Never produces on the **inelastic part** of the demand.

- charges consumers a specific **mark-up** over its marginal cost.
- Has no supply curve, just a **supply rule**.

Mark-up

From the following:

$$MR = MC \Leftrightarrow P(Q) \cdot \left(1 - \frac{1}{|\varepsilon|}\right) = MC \Leftrightarrow \frac{P - MC}{P} = \frac{1}{|\varepsilon|}$$

We get that the markup is:

$$\frac{P - MC}{P}$$

- The markup is never greater than 1 (which means $|\varepsilon| < 1$)
- The more efficient the firm is (lower MC), the bigger the markup is.

Calculation of markup is given demand curve $P(Q)$ and the value of MC:

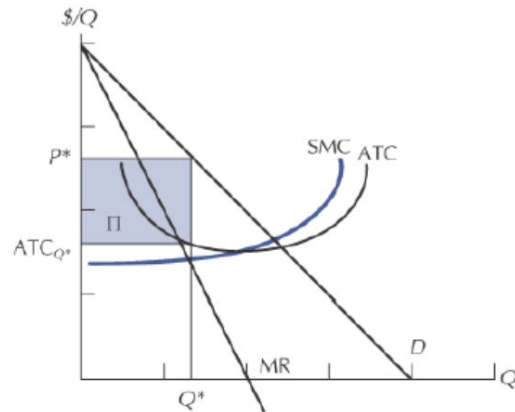
- Find the function MR =

$$\frac{d(P(Q) \cdot Q)}{dQ}$$

- Equalise $MR = MC$ to find the optimal production Q^* .
- Plug it back into $P(Q)$ to find P^* .
- Calculate ε and $1/|\varepsilon|$.

Supply rule

The optimum condition $MR = MC$ yields different (Q^*, P^*) depending on the position of the demand line. For each demand, the monopolist offers only either Q^* or P^* .
[There is no supply curve for monopolies]



Slide 18; Chapter 13 [presentation slides]

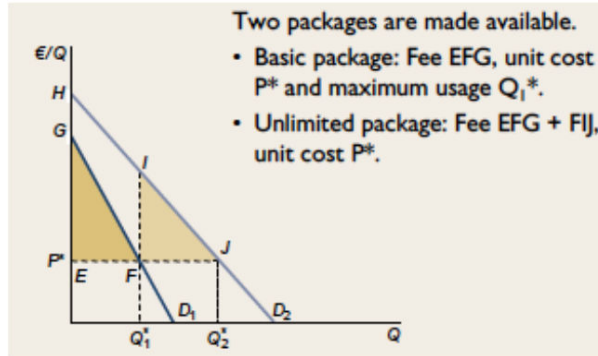
Price discrimination

Monopolies want to make more money than just from simple profit maximisation, so they charge different prices to either different consumers or to the quantity purchased. This technique is called **price discrimination**. Although this term has a negative connotation, it may not always harm all consumers.

First degree price discrimination

By charging different prices based on how much each consumer is willing to pay, the monopolist appropriates all consumer surplus under the demand curve of each consumer. Thus, the marginal revenue and the demand curve are now one and the same. The optimum condition now becomes: $D = MC$.

Even though the monopolist does not know how much a consumer is willing to pay, i.e., which one is **high-demand**, and which one is **low-demand**, they can extract all of everyone's consumer surplus by offering two different two-part tariffs. This means, while extracting as much as possible from the low-demand consumer, it offers the high-demand one a contract that makes them indifferent between that new one or the one of the low-demand consumer. This pushes people to reveal their preferences.



Slide 32; Chapter 13 [presentation slides]

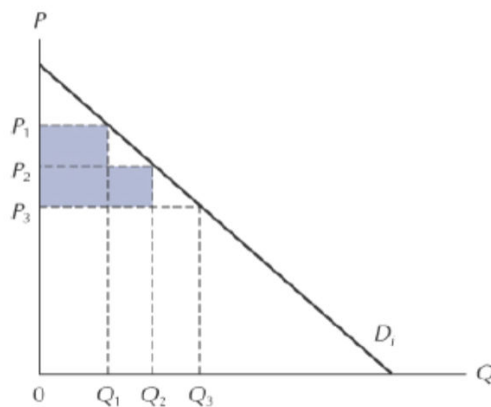
Second degree price discrimination

The monopolist offers not a per unit price but a menu as follows:

For $0 < Q < Q_1$, $P = P_1$;

For $Q_1 < Q < Q_2$, $P = P_2 < P_1$;

For $Q_2 < Q < Q_3$, $P = P_3 < P_2$...



Slide 34; Chapter 13 [presentation slides]

This allows firms to learn information from and extract surplus from consumers.

Second degree price discrimination is a cheaper way to extract surplus from many different consumers.

Third degree price discrimination

Assume the monopolist produces for both markets using a single plant.

Demand on the two markets is given by $P_1(Q_1)$ and $P_2(Q_2)$. The firm's total cost function is given by $C(Q_T) = C(Q_1 + Q_2)$

Remember, profits are simply total revenues – total costs.

Observe that the demand functions given are merely prices as functions of quantity demanded. Thus, we can compute total revenues by multiplying the quantity demanded with their respective prices.

Profits = TR – TC

TC (total cost) is already given by the cost function: $C(Q_T) = C(Q_1 + Q_2)$

TR (total revenue) = Price x Quantity

Price is already given by the demand functions: $[P_1(Q_1)$ and $P_2(Q_2)]$

Thus, to get total revenue, we must multiply the respective quantities of each demand by their prices:

Revenue from market 1: $Q_1 * [P_1(Q_1)]$

Revenue from market 2: $Q_2 * [P_2(Q_2)]$

Total Revenue: $TR = Q_1 * [P_1(Q_1)] + Q_2 * [P_2(Q_2)]$ or

$TR = P_1(Q_1) Q_1 + P_2(Q_2) Q_2 - C(Q_1 + Q_2)$

Now, we can construct the total profits equation by subtracting the total cost from the total revenue:

$$\text{Profits} = P_1(Q_1) Q_1 + P_2(Q_2) Q_2 - C(Q_1 + Q_2)$$

Of course, a monopolist wants to maximise profits. Thus the problem the monopolist faces is:

$$\max P_1(Q_1) Q_1 + P_2(Q_2) Q_2 - C(Q_1 + Q_2)$$

First order conditions are:

$$MR_1(Q_1^*) = MC(Q_T)$$

$$MR_2(Q_2^*) = MC(Q_T)$$

where $MR_i(Q_i) =$

$$\frac{\partial(P_i(Q_i) * Q_i)}{\partial Q_i} \quad \text{with } i = 1, 2$$

The possibility of charging different prices implies that the strategy is more flexible, which in return makes the producer better off. This pricing strategy does not work if markets are identical or very similar.

What if the monopolist cannot sell at a different price on the two markets?

This happens in situations such as when buyers can shop around the two markets? In this case, we need to horizontally sum the two demand lines.

Take for example, two markets with the demand lines:

1. $Q_1 = 100 - P$
2. $Q_2 = 80 - P$

The resulting total market demand function is the horizontal summation of these two markets:

$$Q_t = \begin{cases} 100 - P, & P > 80 \\ 180 - 2P, & 80 \geq P \end{cases}$$

Hurdle model of price discrimination

Discrimination of price based on quality of goods or services. E.g. multiple classes of air travel.

Microeconomics – IBEB – Lecture 11, week 4

Perfect Competition

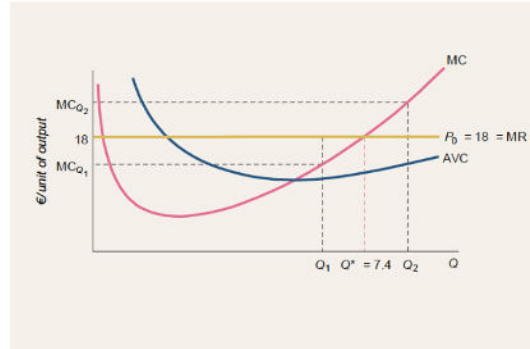
Four conditions of perfect competition

1. The product is **standardised** and is a **perfect substitute** for any other firm's product;
2. Firms are **price takers**: each firm controls such a small share of market that any change in its production does not affect the equilibrium price;
3. **Factors of production** are **perfectly mobile** in the long run – they go to where there are profit opportunities;
4. Firms and consumers have **perfect and complete information**.

Short run

Profit maximisation

Under perfect competition, firms are price takers, so P is given. This implies that $MR=P$, so the **short-run profit maximisation condition** $MR = MC$ changes to: $P = MC$.



Slide 4; Chapter 12 [presentation slides]

Shutdown Condition

- a. If the firm does not produce, its profits are equal to negative fixed costs:

$$\pi(q = 0) = Pq - TC = TR - VC - FC = -FC$$

- b. If it produces, its profits are higher than the negative fixed costs:

$$\pi(q > 0) = Pq - TC = q(P - ATC) = q(P - AVC) - FC > -FC$$

In conclusion, when $P > AVC$, the firm should produce and if $P < AVC$, the firm should completely shut down.

Other conclusions:

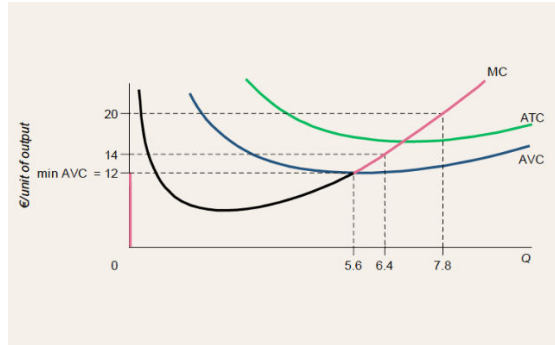
1. $ATC > P \geq AVC$; the firm is making short term losses.
2. $P = ATC$ is the break even point, (no profit).
3. $P > ATC$; the firm is making positive short term profits.

Firm's Supply

When $q > 0$, the firm's supply curve is well-defined and is the solution to:

$$\begin{cases} P > AVC \\ P = MC \end{cases}$$

Graphically, it is the part of the MC curve that is above the AVC curve.



Slide 9; Chapter 12 [presentation slides]

Market supply

If all n firms are identical, we do **horizontal summation**. Industry supply is n times the individual supply curve (a_i is typically negative whereas b_i is positive):

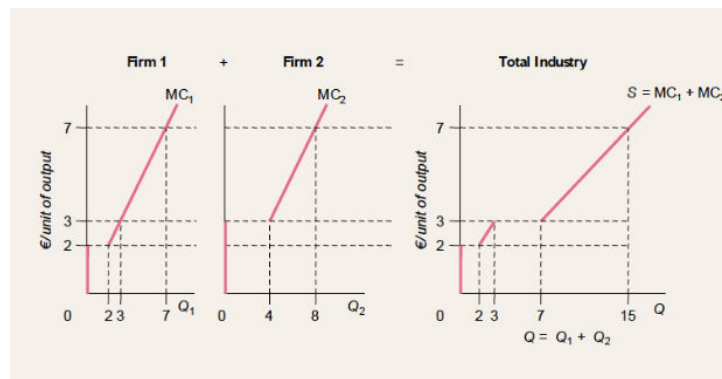
$$\sum MC : Q(= nq_i) = n(a_i + b_i P)$$

Short-run competitive equilibrium

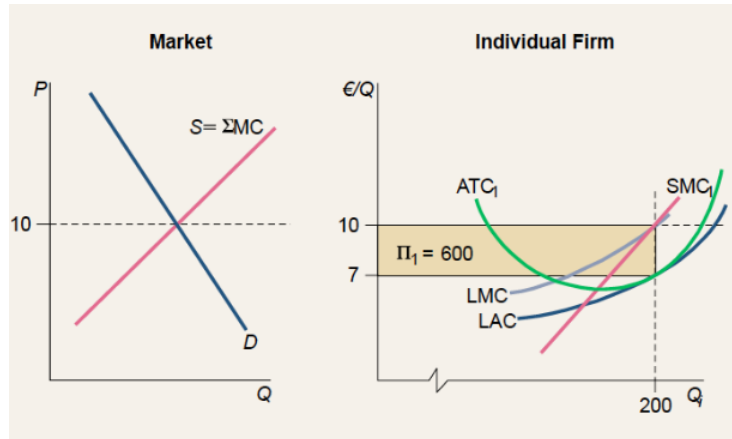
Firms take price P as given so they have an individual **horizontal demand line** that is perfectly elastic at P . The **equilibrium** is at *market supply* = *market demand*:

$$S = D \Rightarrow n(a_i + b_i P) = c - dP, \text{ which implies that } P =$$

$$\frac{c - na_i}{nb_i + d}$$



Slide 12; Chapter 12 [presentation slides]



Slide 14; Chapter 12 [presentation slides]

This equilibrium is **efficient** because the value of the resources given to firms (by consumers) to produce the last unit of output is exactly equal to what firms require to produce that last unit ($P=MC$).

How to compute short-run equilibrium

1. Start with the short-run equilibrium condition of the firm to derive the individual supply.
2. Horizontally sum all the individual supplies to attain the market supply.
3. Equate the market supply to the market demand to find the equilibrium quantity and price.
4. Find how much each firm is going to produce.
 - a. One way to do this is by plotting the equilibrium price in the optimum production condition of each individual firm to recover how much the firm is going to produce.
5. Compute profits.

Long Run

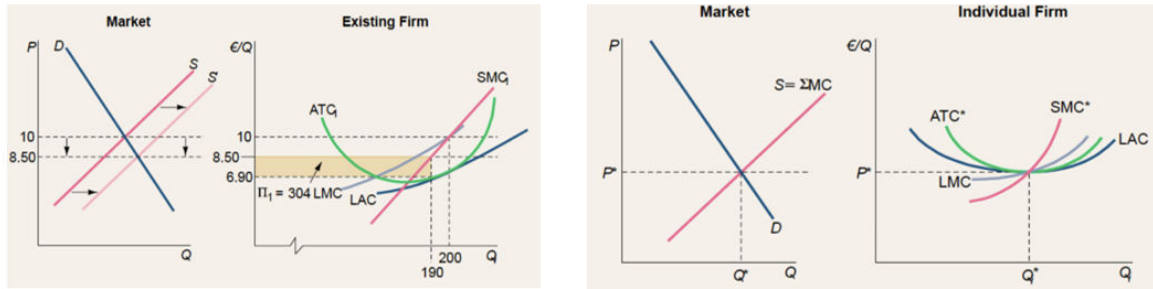
Here, a firm making a **negative profit** goes bankrupt, so the shutdown condition ($P < ATC$) is absolute and final. A firm making a **positive profit** triggers entry of other firms (because the product is standardised, and inputs are perfectly mobile) → supply increases → equilibrium price decreases → firm-level profits decrease.

This requires firms to adjust capital and labour for efficiency. Yet, the profits continue to decrease to 0. The inefficient firms are driven out of the market. The efficient firms

that stay all have the same cost structure (because of the same individual demand), all sell at a single common price and all make zero profits.

For the case of U-shaped LAC, the long run equilibrium is at the minimum of LAC.

The following graphs depict medium and long run adjustments respectively:

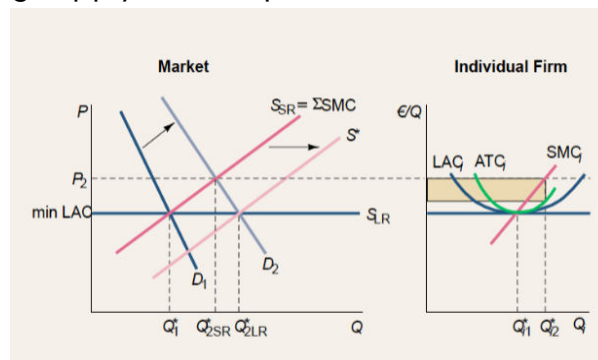


Slides 25 and 26; Chapter 12 [presentation slides]

This equilibrium is **ultra-efficient** because there is no room for further mutually beneficial trades ($P = MC$) and goods are as cheap as possible ($P = \min LAC$).

The long run industry supply

If the input costs rise with the total market production, the LAC curve for each individual firm shifts vertically upwards with the total level of production and this is how an upward sloping supply curve is produced.

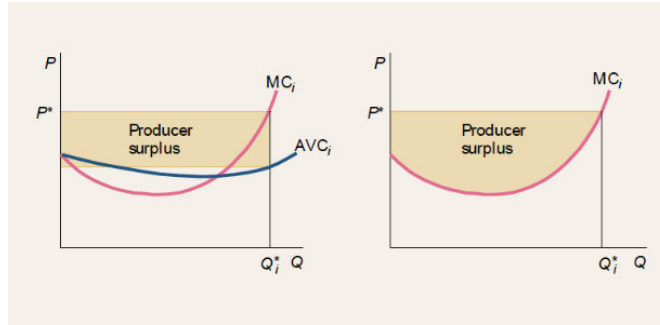


Slide 29; Chapter 12 [presentation slides]

Producer Surplus

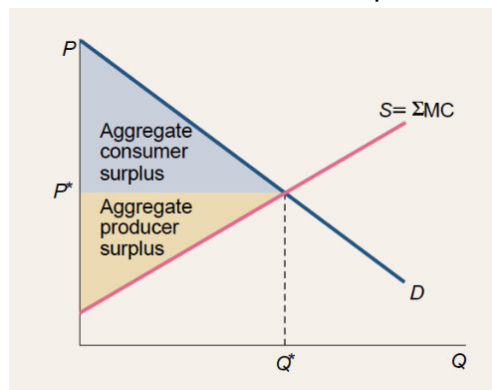
It is the monetary value of the benefit a firm gets from producing. It is given by: $TR - VC$ or $FC + \pi$.

Producer surplus = economic profits only if $FC = 0$. We can represent this in 2 ways:



Slide 32; Chapter 12 [presentation slides]

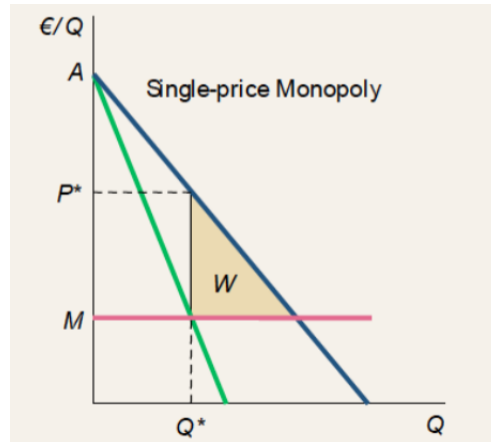
However, we will only compute PS for the whole market, which looks as follows (it is the area above supply curve and under horizontal price line):



Slide 33; Chapter 12 [presentation slides]

Perfect competition vs. Monopoly

Suppose MC of a firm is constant. The source of the inefficiency, triangle W , is known as **Harberger's triangle**. This is a problem of monopoly. Here, first-degree price discrimination is good, because the fact that all surplus is transferred from consumers to producers does not modify the fact that the outcome is as efficient as in perfect competition.



Slide 34; Chapter 12 [presentation slides]

Regulating monopolies

A **standard government policy** is presented by the following steps:

1. Figure out what the monopolist's costs are.
2. Fix the selling price for the monopolist;
3. Let the monopolist produce & sell the quantity it prefers.

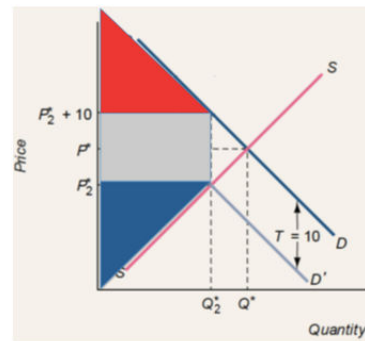
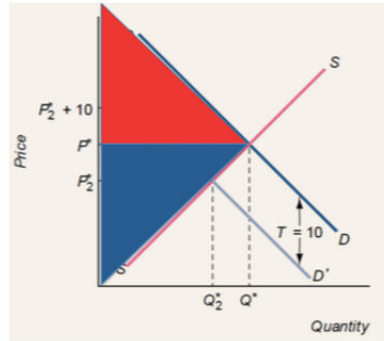
Why is price support a bad idea?

This policy does not allow the market forces to push out the inefficient firms out of the market. The government role in this case should be to help existing firms and their workers transit into new activities. However, this is not easily done.

golden rule for economists: Governments should help economic actors adjust to the forces of well-functioning markets, not interfere with the markets.

Negative efficiency effect of taxes

Suppose a government imposes a unit tax, shifting the demand curve by that amount. A part of the consumer surplus (red) and also of the producer surplus (blue) is now transferred to the government, represented by the grey rectangle.



Slides 39 and 40; Chapter 12 [presentation slides]

However, there is a triangle that goes neither to the producers, consumers or the government. This triangle where efficiency is lost is known as the **deadweight loss**, and it is the reason why a lot of economists are against taxation.

Microeconomics – IBEB – Lecture 13, week 5

Game Theory

What is game theory?

A formal way to study interaction among a group of rational agents who behave strategically.

Keywords:

Group: $n > 1$;

Interaction: how one's actions affect the utility of others;

Strategic: agents take this interdependence into account;

Rational: agents make the best possible choice for them given their objective(s).

Types of Games

We assume players know everything about the game and other players' preferences, i.e. they possess *common knowledge*.

1. Dynamic Games: When dynamics (reputation, credibility, repeated interaction, etc.) are the focus of attention.
2. Static Games: When dynamics are not the focus of attention.

Types of Strategies

Strategies are what each player can do while payoffs are what players get out of the game.

1. Discrete Strategies: These are represented by a matrix and are usually choices between yes or no.
2. Continuous Strategies: These are represented by writing down each player's optimization level. E.g: optimal production level.

Representing a Static Game

This representation includes:

- a. The players
- b. The strategies per player
- c. The payoffs per player

| | | Player 2 | |
|----------|------------|------------|------------|
| | | Strategy A | Strategy B |
| Player 1 | Strategy 1 | a, b | c, d |
| | Strategy 2 | e, f | g, h |
| | Strategy 3 | i, j | k, l |

Slide 8; Chapter 3[presentation slides]

This table is a matrix where the first number in each cell is player 1's payoff; the second number is player 2's.

Assumption: complete information and common knowledge.

Representing a Dynamic Game

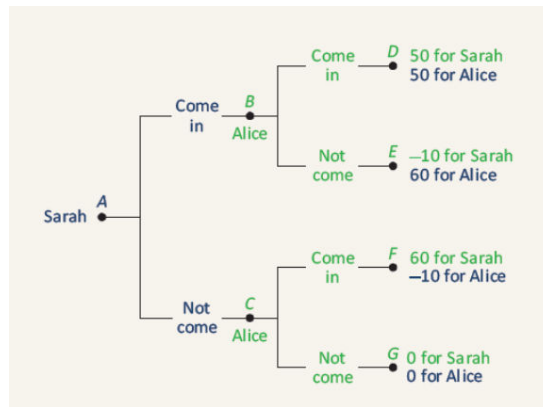
Use extensive form representation, usually a tree, indicating:

- a. The players
- b. The strategies per player
- c. The payoffs per player

d. Player Function

In this case, strategies are now **vectors**. This is because you may be asked to play more than once and your strategy needs to specify what you will do in each case. The **size of a vector** is the number of contingencies each player has. Therefore, to attain the strategy, one must first count the number of times a player is asked to play.

Example:



Slide 10; Chapter 3 [Presentation Slides]

Here, Sarah moves once.

Strategy: one action

Strategy set: two actions (come in/not come)

Alice has two subgames.

Strategy: Pairs of actions following Sarah's move

Strategy set: 4 strategies (come in, come in), (come in, not come), (not come, come in), (not come, not come)

Sub-games

Remaining part of the **total** game, starting at a point where a player needs to make a decision. They are a key concept in identifying equilibria of dynamic games.

Normal form representation of dynamic games

Dynamic games in which the player's strategies are discrete can be represented in **matrix form**. After pinning down each player's strategy, write down a matrix in which the number of columns/rows corresponds with the number of strategies each player has.

| | | | | | |
|-------|----|---------|--------|---------|---------|
| | | Alice | | | |
| | | C, C | C, NC | NC,C | NC, NC |
| Sarah | C | 50, 50 | 50, 50 | -10, 60 | -10, 60 |
| | NC | 60, -10 | 0, 0 | 60,-10 | 0, 0 |

Slide 18; Chapter 3 [Presentation Slides]

Famous Examples

Static game with discrete strategies

1. Prisoner's Dilemma

Two criminals are interrogated in separate rooms. Police want them to confess their guilt and criminals want the least years in jail. The matrix outcomes of their options is:

| | | | |
|----------|---------|----------|---------|
| | | Player 2 | |
| | | Deny | Confess |
| Player 1 | Deny | b, b | d, a |
| | Confess | a, d | c, c |

Slide 20; Chapter 3 [Presentation Slides]

with $a > b > c > d$.

For their own self-interests, both criminals will confess, which is what police want.

2. Stag Hunt

A group of friends go hunting. If they want to hunt a stag, all of them need to cooperate. If a single one of them deviates, each of them can only get a hare.

| | | |
|------|---------|--------|
| | Stag | Hare |
| Stag | $a, a,$ | d, c |
| Hare | c, d | b, b |

Slide 21; Chapter 3 [Presentation Slides]

with $a > b > c > d$.

This game can also be used to explain arms races.

3. James Dean

Two people face each other in a duel. Each can choose to defy the other or concede defeat. If they defy each other, they both die. If one concedes and the

other defies, the defying player is the winner. If they concede, they both go home in shame.

| | | |
|---------|----------|---------|
| | Defy | Concede |
| Defy | -10, -10 | 5, -1 |
| Concede | -1, 5 | 0, 0 |

Slide 22; Chapter 3 [Presentation Slides]

4. The Battle of the Sexes

Going out together, you would prefer a restaurant whereas your partner prefers a bar. You have to choose where to go without being able to talk to each other.

| | | |
|------------|------------|--------|
| | Restaurant | Bar |
| Restaurant | 10, 5 | -1, -1 |
| Bar | -1, -1 | 5, 10 |

Slide 23; Chapter 3 [Presentation Slides]

Static game with continuous strategies

1. COURNOT MODEL OF DUOPOLY COMPETITION

Two firms both produce and sell identical goods on the same market. The market demand for the good is:

$$P = a - b (q_1 + q_2)$$

The two firms compete for customers, who view the two goods as 1-to-1 perfect substitutes and wish to maximise their profits. They compete through their choice of the *quantity they produce*. This should not be represented in a matrix or else the said matrix will be infinite. Each firm's profit is then:

$$\pi_i = P(Q) q_i - TC(q_i) = [a - b(q_1 + q_2)] q_i - TC(q_i)$$

2. BERTRAND MODEL OF DUOPOLY COMPETITION

Two firms both produce and sell identical goods on the same market. The market demand for the good is:

$$P = a - b (q_1 + q_2)$$

The two firms compete for customers, who view the two goods as 1-to-1 perfect substitutes, and wish to maximise their profits. They compete through their choice of the *selling price*.

Simplifying assumption: the two firms have the same constant marginal cost of production c .

3. HOTELLING'S LINEAR MODEL OF COMPETITION

Markets in which firms compete for customers when customers want to go to the firm that is closest to them. In this model, there is one good with a fixed price "p". All consumers want to purchase only one good.

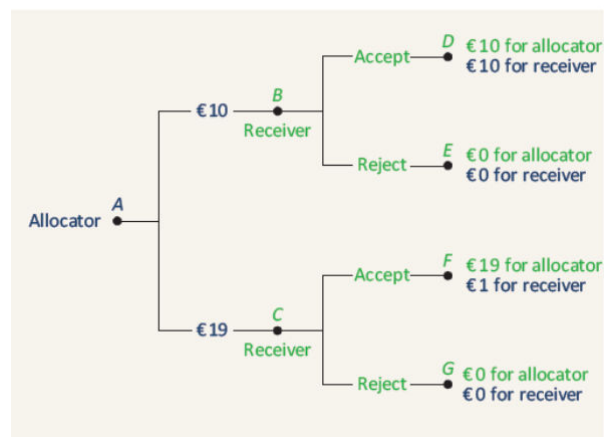
This model attempts to answer questions such as where sellers will locate their shop and how does this answer depend on the amount of sellers?

4. SALOP'S CIRCULAR MODEL OF COMPETITION

This is the same as Hotelling's but now, customers are uniformly distributed on a unit circle.

Dynamic game with discrete strategies

1. THE ULTIMATUM GAME



Slide 30; Chapter 3 [Presentation Slides]

- The strategies of the Receiver are vectors.
- The strategy of the Allocator is merely 1 action.

2. THE REPEATED PRISONER'S DILEMMA

It is the same as Prisoner's Dilemma but the game is repeated n times (n can be finite or infinite), with the probability p of the game being played again.

Note: A game played many times is also called a stage game.

3. THE REPEATED GROCERY STORE GAME

Consider an individual who wishes to buy a banana every day during their stay at a touristic place. Their willingness to pay is X euros, while the store can

set the price either at high or at low ($0 < P_L < P_H < X$). We can use this game to rationalise why tourists face higher prices than locals.

| | | |
|-------|----------------|---------|
| | Buy | Not buy |
| P_H | $P_H, X - P_H$ | 0, 0 |
| P_L | $P_L, X - P_L$ | 0, 0 |

Slide 32; Chapter 3 [Presentation Slides]

4. BARGAINING OVER A BAG OF GOLD COINS

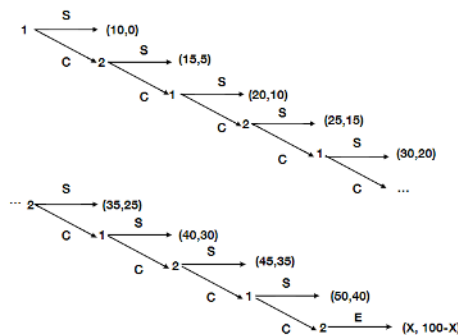
Pirates need to share a bag of 100 coins. Coins are not divisible. The first one to make a proposal is the senior pirate. The decisions are made by a strict majority rule. In case of tie, the proposer is pivotal. If the majority rejects the proposal, the proposal is thrown overboard and dies. The next person to propose is then the next senior pirate. This continues until the proposal is accepted or until only one pirate is left.

Payoffs:

- 1) Each pirate wants to survive;
- 2) Everyone wants to maximise the number of coins they receive;
- 3) Each pirate would prefer to throw another overboard;
- 4) They do not trust each other.

5. CENTIPEDE GAME

Two people take turns at either stopping or continuing the game. If they continue, the prize money increases by 10e. In each round, the player who has the move can stop the game and earn a higher payoff than the other. If the game reaches 100e, the game ends with two players sharing 100e, with player 1 getting X euros.



Slide 34; Chapter 3 [Presentation Slides]

Dynamic Games with continuous strategies

1. STACKELBERG MODEL OF DUOPOLY COMPETITION

Identical to Cournot game but the **leader firm** produces its output before the **follower firm** decides how much to produce.

1. Leader chooses q_L ;
2. Follower observes q_L and then chooses q_F ;
3. Product is sold at the market-determined price, firms receive their profits.

Equilibrium

Strict dominance

A strategy that pays less regardless of what other players do. We consider a strategy to be dominated only if you can always do better by playing another.

Iterated elimination of strictly dominated strategies (IESDS)

Deleting iteratively and in turn one strictly dominated strategy per player until no such strategies remain to find the equilibrium.

Note: the order of deletion of strategies does not matter. Each action in each set of player actions that resists this iterated elimination is said to be rationalizable.

Example:

| | | Player 2 | | |
|----------|---|----------|-------|------|
| | | L | M | R |
| Player 1 | A | 0,1 | 1,1 | -5,2 |
| | B | -4,8 | 2,5 | 10,6 |
| | C | 6,5 | 10,-1 | 6,5 |
| | D | 6,5 | 5,4 | 6,5 |
| | E | -3,5 | 2,-3 | 1,2 |

Slide 42; Chapter 3 [Presentation Slides]

Comparing the numbers corresponding to each player's payoffs, we see that for player 1, A is strictly dominated by C because $0 < 6$, $1 < 10$, $-5 < 6$, so we delete row A. For player 2, M is strictly dominated by R because $1 < 2$, $5 < 6$, $-1 < 5$, $4 < 5$, $-3 < 2$, so we delete column M etc.

Keep both strategies if you are indifferent between the two, and do not delete strategies when they are weakly dominated.

Strictly Dominated strategies help us solve games such as the Prisoner's Dilemma, but have limitations that prevent it from solving games like The Battle of The Sexes, James Dean, etc.

Microeconomics – IBEB – Lecture 14, week 6

Maximin

Among the worst possible outcomes, each player chooses the one with the highest payoff to minimise damage.

Example:

| | | |
|---|--------|--------|
| | L | R |
| U | 3, ... | 0, ... |
| D | 1, ... | 2, ... |

For U, 0 is the worse outcome ($0 < 3$) and for D, 1 is the worse outcome ($1 < 2$).

Comparing 0 and 1, we choose D to lessen the possible loss.

Nash equilibrium

When IESDS cannot precisely predict the equilibrium, we use **Nash equilibrium (NE)**.

An NE is a **vector**, a collection of strategies $(a_1^*, a_2^*, \dots, a_i^*, \dots, a_n^*)$ such that the strategy a_i^* you play is optimal for you given that other players are playing according to $(a_1^*, a_2^*, \dots, a_n^*)$. We fix what we think the other player will be doing, and given this expectation, we choose the best action to do.

Here, what you think others do is truly what they do (**rational expectation**).

Finding NE:

1. Fix what the others are doing, $\bar{a}_2, \dots, \bar{a}_i, \dots, \bar{a}_n$ and compute your own best choice.
2. Repeat for each player i .

Player i 's best choice(s) of a_i in reply to the other players playing $(\bar{a}_2, \dots, \bar{a}_i, \dots, \bar{a}_n)$ is called player i 's best response. Best responses $BR_i(a_1, \dots, a_n)$ can be a single action or function.

3. NE is the best response of everyone.

In a matrix, each player's best response is highlighted by underlying payoffs. In games with continuous objective functions, to find the best responses, we solve each player's problem and then the system of BRs – which is a system of n equations in n unknowns, $a_1^*, a_2^*, \dots, a_n^*$.

Examples:

| | | Criminal 2 | |
|------------|---------|--------------|---------------------|
| | | Deny | Confess |
| Criminal 1 | Deny | b, b | d, <u>a</u> |
| | Confess | <u>a</u> , d | <u>c</u> , <u>c</u> |

Where $a > b > c > d$

Choose the cell that has both responses underlined. Cells without underline hint at a lack of mutual consistency and thus are not the answer. The Nash equilibrium is by strategies and not payoffs.

NE = (Confess, Confess)

Outcome: Both players select the option to confess.

| | Restaurant | Bar |
|------------|----------------------|----------------------|
| Restaurant | <u>10</u> , <u>5</u> | -1, -1 |
| Bar | -1, -1 | <u>5</u> , <u>10</u> |

Here, the nash equilibrium is (Restaurant, Restaurant), (Bar, Bar)

Subgame perfect equilibrium (SPE)

An SPE is an NE with an **extra condition** that we must plan complete strategies for every contingency.

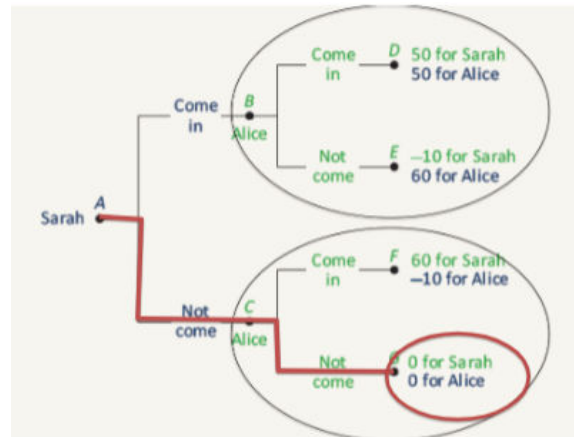
That is, a collection of strategies $(a_1^*, a_2^*, \dots, a_n^*)$ such that, for any player i , playing the strategy prescribed by a_i^* in each of the player's subgames yields them the highest payoff among all feasible strategies for them, *given that the other players are playing according to* $(a_1^*, a_2^*, \dots, a_n^*)$.

How to find SPE?

Use backward induction. To decide what one player should do, that player must form correct expectations about the best responses of the following players.

1. Count the number of contingencies.
2. Count the number of subgames.
3. Find the optimal behaviour in the subgames by backward induction.

Sarah and Alice example:



Slide 80; Chapter 3 [Presentation Slides]

Depending on Sarah's choice, Alice can end up at either B or C. At B, Alice can have payoff 50 or 60. She will choose 60 and not come. At C, Alice can have payoff -10 or 0. She will choose 0 and not come. Knowing this, Sarah has two options: payoff -10 at E or 0 at G. She will choose 0 and to get 0, she will not come.

The perfect subgame equilibrium is:

1. Sarah does not come in.
2. Alice does not come in if Sarah does not come in, and Alice does not come in if Sarah comes in.

Thus:

SPE = (Not come; (Not come; Not come)) or
(C; (E; G))

where, **C is Sarah's choice of not going;**

E and G are Alice's choices of not going.

The first NC is Sarah's strategy; the two NCs between brackets are what Alice will do in each of her contingencies.

Credibility of promises/threats

Promises/threats are credible if they are part of a subgame perfect equilibrium. In the above example, Sarah should not believe it when Alice says: "If you come in, I come in too" because when Sarah actually comes in, Alice would rather go for the higher payoff of not coming than keeping the promise.

General Equilibrium and Market Efficiency

So far, we have only dealt with **partial equilibrium analysis**, which is analysing the behaviour of a market or an agent in isolation from the rest of the economy. For example, regarding how much of a good a consumer wants to buy, we assumed that its price was fixed and that any of its quantity was available at that price. However, there are times when the desired quantity is not available and the price you get is affected by other consumers who want that good as well. The analysis that takes this cross effect into account is **general equilibrium analysis (GEA)**, which is typically concerned with finding **equilibrium prices**. We thus focus on the second key role of prices – their allocative role.

Part I: simple exchange closed economy

Characteristics of a **closed economy**:

- The agents own all the resources in that economy;
- Production and consumption are within the same economy;
- no exchanges with the rest of the world.

GEA of a simple exchange economy

In such an economy, there is no production, only consumers, who are endowed initially with a share of the economy's resources. Assume an economy with 2 consumers, A and B, and 2 consumption goods only, clothing C and food F:

$$C_B = C - C_A; F_B = F - F_A$$

Whatever B has is a subtraction of total goods minus what A owns.

C_B is the *initial* quantity of clothing that B has,

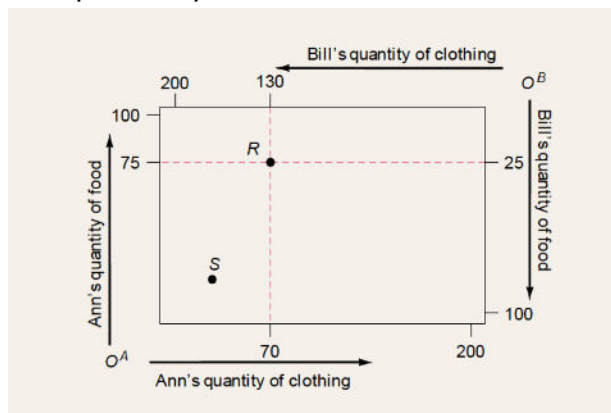
C_A is the *initial* quantity of clothing that A has,

F_A is the *initial* quantity of food that A has,

F_B is the *initial* quantity of food that B has.

These values are initial because nothing guarantees that B/A wants to consume exactly the quantity of clothing and food that s/he has in his/her pocket. Thus, B and A are going to start interacting to find a deal to exchange clothing and food between the two of them.

Using the **Edgeworth Box** to represent this, we have the height and width of the box corresponding to the total available quantity of each of two goods. The two consumers are placed respectively in the bottom left and the top right corner.



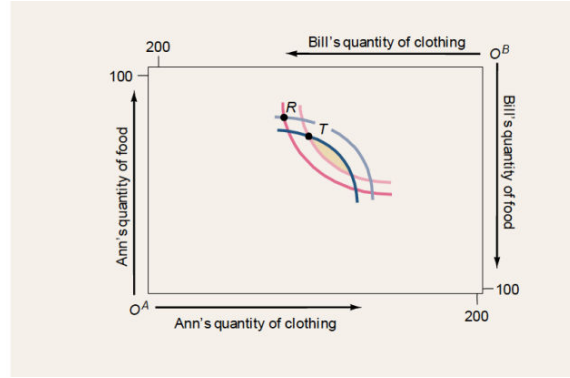
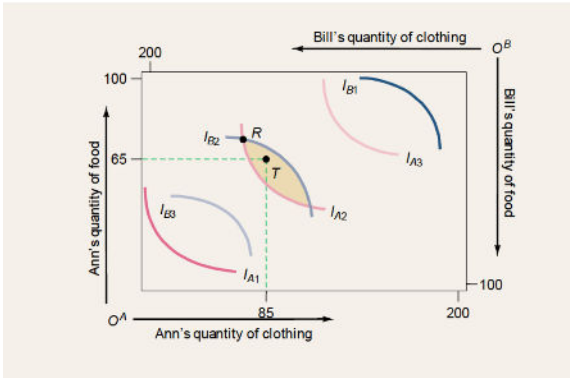
Slide 10; Chapter 17 [Presentation Slides]

Exchange and contract curve

Both consumers' preferences can be represented by **indifference curves** (the two consumers' utility if they consume their endowment) with following characteristics:

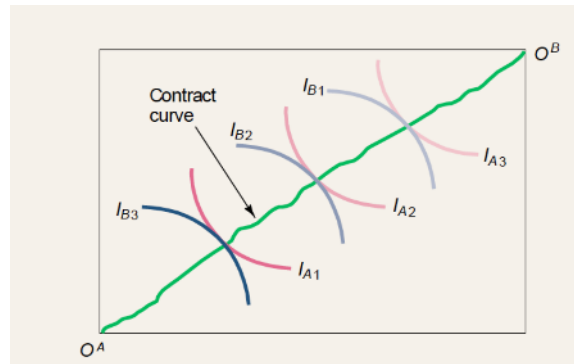
| | Consumer A | Consumer B |
|--|--------------------|------------------|
| Origin | Bottom left corner | Top right corner |
| Convex or Concave | Convex | Concave |
| Direction of increasing Utility | ↗ | ↙ |

For both, the further away the utility function is from the origin, the better.



Slide 14 and 16; Chapter 17 [Presentation Slides]

In the above figures, R is the initial endowment point and T is the point where two consumers trade and can both be better off. They will continue to exchange until neither of them can increase utility without decreasing that of the other agent. That is point M – the tangency of two consumers' highest indifference curves. This point is also **Pareto efficient**. The set of Pareto optimal points, representing efficiency in consumption, is the **Contract curve**, as follows:



Slide 18 and 20; Chapter 17 [Presentation Slides]

Given the endowments, only a portion of the contract curve is attainable. Therefore, the **equation of Contract curve** is the solution to:

$$MRS_A = MRS_B \text{ subject to } C_B = C - C_A; F_B = F - F_A$$

Microeconomics - IBEB - Lecture 15, week 6

Adding relative prices

Assume there is an **auctioneer** who fixes the prices such that:

$$P_C/P_F = 1;$$

$$C_B = 50 = C_A;$$

$$F_B = 100 = F_A;$$

Draw a line with slope -1 that goes through initial endowment point E, indicating the space of possible trades. Applying the **rational choice model**, we can find the preferred consumption bundle of A and B on this line. Solving each consumer's respective problem yields points A^* and B^* . However, these two points are not equilibrium, because there is excess demand for clothing and excess supply of food.

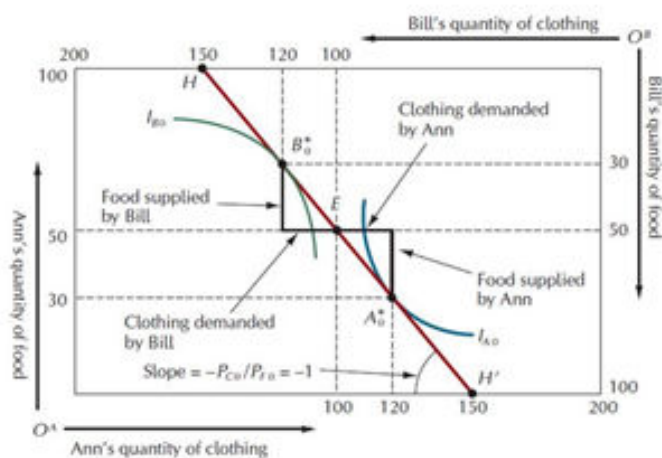


Figure 17.9, chapter 17, page 528 - Microeconomics and Behaviour, 2nd edition, R. Frank and E. Cartwright (2016)

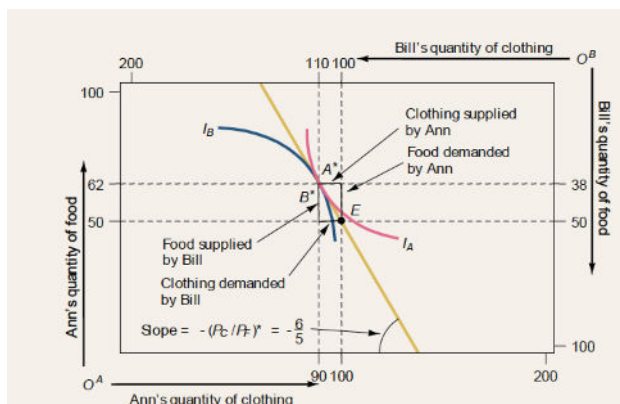
In order to reach equilibrium, we have to change the relative prices, making clothing more expensive with respect to food, until the optimal points for A and B coincide. At that point, $C_A = C_B$ and $F_A = F_B$. This condition is called **market clearing**.

Walras' Law: in a general equilibrium world with n markets (goods/ services), if $n-1$ markets are in equilibrium, this also holds for the n th market.

In conclusion, in equilibrium, there exist P_C and P_F such that:

$$C_A^* = C - C_B^*; F_A^* = F - F_B^*;$$

$$MRS_A = MRS_B = P_C^* / P_F^*$$



Slide 34; Chapter 17 [Presentation Slides]

Finding equilibrium relative prices

In the example where we're given the utility functions of two consumers ($i = 1, 2$): $x_{11}^\alpha x_{21}^{1-\alpha}$ and $x_{12}^\beta x_{22}^{1-\beta}$ and their endowments: (1,2) and (3,4). We do the following:

1. **Determine consumers' wealth**

Consumer 1: $p_1 + 2p_2$

Consumer 2: $3p_1 + 4p_2$

(where p_1 and p_2 are the prices of goods 1 and 2 respectively).

2. **Find demand for two goods by solving the Lagrange for each consumer**

(same method as in rational consumer choice problem)

The Lagrange function for consumer 1 in this example:

$$L_1 = \alpha \ln x_{11} + (1 - \alpha) \ln x_{21} - \lambda [p_1 x_{11} + p_2 x_{21} - p_1 - 2p_2]$$

Solve for consumer 2 too to arrive at $x_{12}, x_{22}, x_{21}, x_{11}$ as functions of p_1 and p_2 .

3. **Do market clearing for either of the goods** (equilibrium prices must be such that total demand of two consumers for one good equals their total endowments for that good)

$$x_{11} + x_{12} = 1 + 3 = 4 \text{ or } x_{21} + x_{22} = 2 + 4 = 6$$

4. Plug in the values (found in step 2) to step 3 to **find the equilibrium price ratio** (tip: normalise one of the prices to 1).

$$\frac{p_1^*}{p_2^*} = \frac{2\alpha + 4\beta}{4 - \alpha - 3\beta}$$

Welfare economics – theorems

First fundamental theorem (Smith's invisible hand theorem)

"An equilibrium produced by fully-functioning competitive markets (through the appropriate pricing of the goods considered) will exhaust all possible gains from exchange."

So, in competitive markets there exists a **relative price** which makes equilibrium consumption allocation **Pareto optimal** (all equilibria are on **contract curve**). Markets push consumers to reach an equilibrium in which everyone is better off and nobody can be made further better off without at least one person in the economy becoming worse off.

Second fundamental theorem

"Any allocation on the contract curve can be sustained as a competitive equilibrium."

In other words:

ISSUE OF EQUITY \neq ISSUE OF EFFICIENCY IN ALLOCATION

Thus, the government should not interfere with the market. Rather, they may instead change the initial endowment and let the market do its work.

Microeconomics – IBEB – Lecture 16, week 6

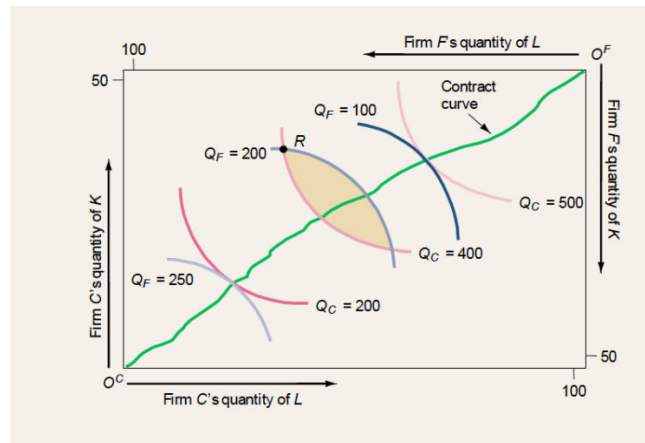
This lecture continues with the General Equilibrium theory. To start, we take a few assumptions, namely: That the economy is closed for now, and that two firms produce one of the two goods as their only production input. We also assume that the economy is characterized by a quantity of labor and a quantity of capital that the two firms own and must share. Finally, suppose firms are price takers on the markets for inputs, meaning they take "w" and "r" as given.

Part II: GEA with production

Efficiency in production with two inputs in a closed economy

Suppose 2 firms produce *food* and *clothing* respectively using *labour* (L) and *capital* (K). They take P of labour (w) and P of capital (r) as given.

How to use the inputs optimally?



Slide 46; Chapter 17 [Presentation Slides]

In the above figure, the convex curves are **isoquants**, so optimality requires:

$$MRTS_i \equiv \frac{MPL_i}{MPK_i} = \frac{w}{r}, \text{ with } i = C, K$$

This condition is most easily interpreted when written as:

$$\frac{MRL_i}{w} = \frac{MRK_i}{r}$$

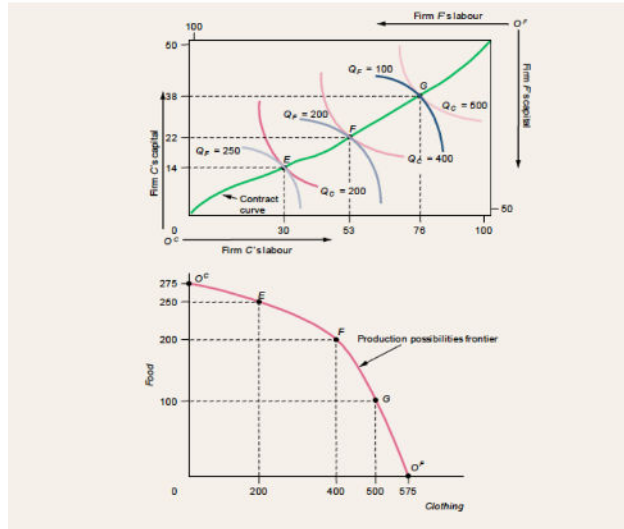
Here the equilibrium is efficient in terms of the allocation of factors of production.

Efficiency in product mix

In the above example, we talked about all the points that are considered efficient but we still need to figure out which production point would be selected in equilibrium.

So, the key question is: *Is the equilibrium product mix efficient?*

The answer lies in the **production possibilities frontier** (this is the translation of the production contract curve).



Slide 50; Chapter 17 [Presentation Slides]

The slope of it is the **Marginal Rate of Transformation (MRT)**, i.e. how much an extra unit of x (in this case clothing) costs in terms of units of y (in this case food), through a firm-to-firm exchange of L and K along the contract curve.

The condition to choose a production point in equilibrium:

$$\text{MRT} = \left. \frac{dF}{dC} \right|_{\text{PPF}} = - \frac{MC_C}{MC_F} = - \frac{P_C^*}{P_F^*}$$

This condition also ensures that the product mix is efficient because the two goods' relative price is equal to the two goods' relative marginal cost.

How much output is expected to be sold to the markets?

Consider the following: owners of L and K are also the consumers A and B:

A owns L_A and K_A ; B owns $L_B = L - L_A$ and $K_B = K - K_A$

Suppose there is perfect competition – all firm revenues are used to pay production inputs. Firms reward owners of two inputs at (given) competitive prices w and r . The two consumers' ownership shares of labor and capital together with their prices determine the two consumers' income:

$$M_i = wL_i + rK_i \text{ where } i = A, B \text{ with } L_A + L_B = L \text{ and } K_A + K_B = K$$

This income is spent on two goods following usual consumption rule:

Conclusions:

1. The **product mix** is efficient because:

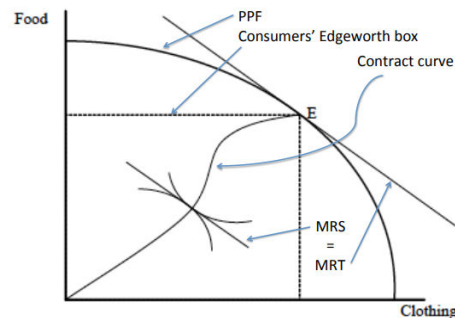
$$\frac{P_C^*}{P_F^*} = MRT$$

2. Given endowments, **consumption mix** is efficient:

$$(MRS_A = -\frac{P_C^*}{P_F^*} = MRS_B)$$

3. The economy's **total output** of each good is exactly **equal** to the economy's **total demand** for the same good:

$$(C = C_A^* + C_B^*; F = F_A^* + F_B^*)$$



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Based on the GE theory, we can see the power of the market – firms do not talk to or coordinate with consumers, yet their choices are mutually consistent.

Issue of equity

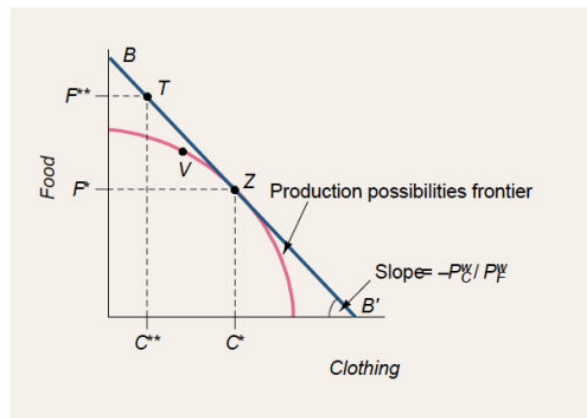
Even when markets work well and the general equilibrium is fully efficient, it may be unequal. **Policy intervention** should change the endowments of consumers/owners (generating as little deadweight loss as possible) but then let markets do their job. This should be one of the central goals of a nation's tax code.

Opening up to trade

In the following model, consumers and producers take prices of goods as those in the world (P_C^W, P_F^W). Trade with the rest of the world is always possible. World relative prices mean that an economy should produce a mix such that:

$$-\frac{P_C^W}{P_F^W} = MRT = MRS_i \text{ for } i = A, B$$

International trade allows a country to choose its most preferred consumption bundle on the international budget constraint, so the economy as a whole is now better off.



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Distributional issues

General equilibrium analysis says that trade can give globally more of everything, but not that every consumer is better off.

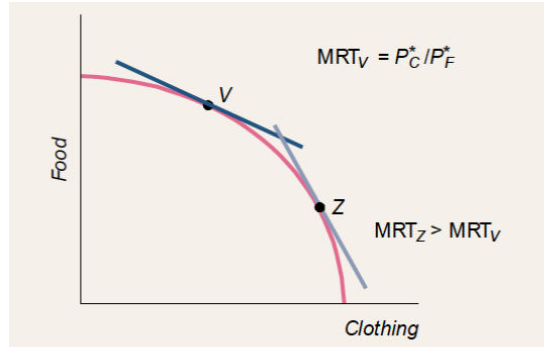
Initial endowments (i.e. the employment sector) will determine whether or not trade makes you better off. For example, if clothing has higher demand and it uses more K than L, then owners of K are better off at the expense of owners of L.

Thus, intervention must be about soothing the transition, not preventing it.

Taxes in GE

Taxes on final goods do not affect the allocation of endowments of consumption goods efficiency, nor the allocation of the factors of production. However, they do lead to an inefficiency in the production mix:

$$MRT = - \frac{(1 - t_C)P_C^*}{(1 - t_F)P_F^*} \neq - \frac{P_C^*}{P_F^*} = MRS$$



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In order to not distort the efficient allocation, taxes must not modify relative prices. In this case, taxes are called **lump-sum**. This means that they need to be of the form $M-t$ and not $(1-t)p$.

When does the GE framework break down?

1. Monopoly: Prices are higher than the efficient level.
MRS of some consumers is greater than MRT, so the production mix is inefficient.
2. Externalities:
Can have consumption, production externalities or both.
3. Uncertainty and Imperfect Information:
Leads to adverse selection and moral hazard.

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Imperfect Competition

Cournot model of duopoly competition

Here, 2 firms produce an identical product that they both sell at the same market, and they compete to maximise profits through their choice of quantity to produce.

This Market is still a downward sloping line. Assume that both goods are 1-to-1 perfect substitutes.

- **Market demand curve:** $P = a - b(q_1 + q_2)$
 - o Two firms compete for customers on this market. They compete through their quantities produced.
- **Each firm's cost function:** $TC_i(q_i) = f_i + c_i q_i$
- **Each firm's profit:** $\pi_i = P(Q)q_i - TC(q_i) = [a - b(q_1 + q_2)]q_i - TC(q_i)$
 - o Each firm still seeks to maximise their profit.

There is one strictly dominated strategy, which is producing more than $q^{monopoly}$. In case other firms drop out, the remaining firm would be a monopoly. Because demand is downward sloping, producing any quantity above that of a monopoly would reduce profits. Thus, each firm must never produce more than the monopoly quantity.

The limit a firm would be willing to produce is $q^{monopoly}$. Anything above this is a strictly dominated strategy.

Solving for the Nash Equilibrium of the Cournot game

To solve for the Nash Equilibrium of the Cournot game, we must:

1. Write down the objective function of each player (in this case, it is the profit function).
2. Maximise this function
 - a. The outcome of this maximisation is typically a function of what the other players are doing. These are the best responses of the game.
3. Make sure the best responses are mutually consistent by imposing the rational expectation condition.
 - a. This imposition yields a system of equations
4. Solve the system of equations
 - a. The result is the Nash Equilibrium.

This game is one with continuous strategies. So, we must first solve each firm's problem and solve the system of resulting best responses.

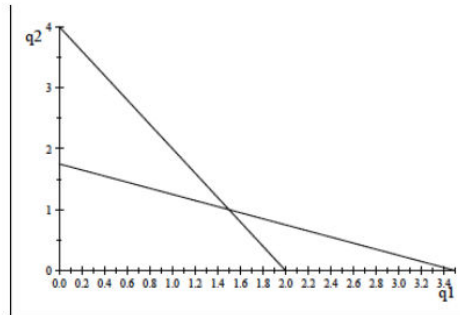
Let us first discover each firm's problem. To do this, we must first write the profit function of the firm. The profit function is the same as the firm's problem. Thus, the firm's problem is:

$$\text{Max}_{q_i} [a - b(q_i + \bar{q}_j) - c_i] - f_i$$

We arrive at the NE:

$$q_1^* = q_2^* = \frac{a-c}{3b} \quad (a > c)$$

This is the intersection of the two best response curves:



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Bertrand model of duopoly competition

In the Bertrand model, firms compete through their prices. In this model, we assume that:

- There is no difference between short and long run
- Two firms have the same constant marginal cost of production c .

Is there a strictly dominated strategy?

Yes, and it is selling at $P < c$ because in this case, the firm goes bankrupt.

What is the Nash Equilibrium in this case?

NE is both firms sell at $P = c$. This is relatively similar to perfect competition, except that the price is not given, but such that maximises both firms' profits.

Thus, the number of firms does not determine the degree of competitive pressure on the outcome of the market in terms of welfare for society.

Cournot vs. Bertrand vs. monopoly

Calculations of price under different competition give:

1. Cournot:

$$Q = 2 \frac{a-c}{3b} \rightarrow P = a - b \frac{2(a-c)}{3b} = \frac{a}{3} + \frac{2c}{3}$$

2. Bertrand: $P = c$

3. Monopoly:

$$Q = \frac{a-c}{2b} \rightarrow P = \frac{a}{2} + c$$

With $a > c$, we have the following ranking of markets in terms of efficiency:

1. Bertrand and Perfect Competition
2. Cournot
3. Monopoly

The Stackelberg model

This model is the same as Cournot except that there is timing of the game. Now, firms can see what the other firm has done before deciding what to do. This is for games whose firm movement is no longer considered simultaneous.

The firm that moves first in the game is called a leader. This firm is typically better off.

- **t=1:** Leader (firm 1) chooses q_L ;
- **t=2:** Follower (firm 2) observes q_L and then chooses q_F ;
- **t=3:** Products are sold on the market at the market-determined price;
- **t=4:** Firms receive their profits and game ends.

We solve the problem backwards (backward induction), by first analysing what firm 2 chooses following the choice of firm 1. Firm 2's problem:

$\max \{q_2[a - b(q_2 + q_1^*) - c_2]\}$ yields:

$$q_2^* = \frac{a - bq_1^* - c_2}{2b}$$

Insert firm 2's BR function into firm 1's problem: $\max \{q_1[a - b(q_1 + q_2(q_1)) - c_1]\}$ to get max which yields the SP outcome as follows:

$$q_1^* = \frac{a + c_2 - 2c_1}{2b}$$

$$q_2^* = \frac{a + 2c_1 - 3c_2}{4b}$$

The SP equilibrium is thus:

$$(q_1^* = \frac{a + c_2 - 2c_1}{2b}; q_2(q_1) = \frac{a - bq_1 - c_2}{2b})$$

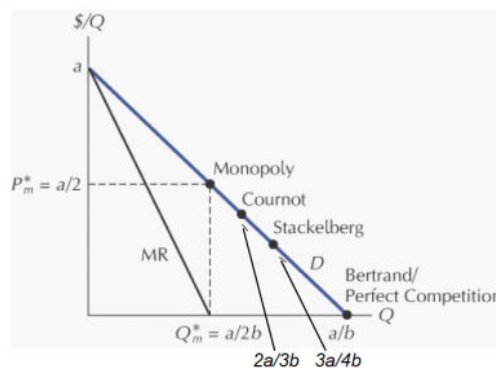
Important note: check that profit of firm 1 is higher than of firm 2.

Stackelberg vs. Cournot

If the firm can choose between a game in which everyone is equal and one in which it has an advantage, it will choose the second option. The leading firm would rather be a monopolist to a Stackelberg Leader but would rather be a Stackelberg Leader to a Cournot duopolist.

Comparison of market outcomes

We represent the total industry output graphically through the following, allowing us to compare outcomes of different oligopoly structures:



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From this graph, we can conclude that, if they could, firms would **collude** and act as a single monopolist in order to maximize profits. This is known as the **cartel theory**.

In reality, we do not treat firms symmetrically. Therefore, the previous models have certain limitations. We need to consider **spatial competition** because firms also choose the location where they will position themselves.

Hotelling's line

This model emphasizes the **strategic interdependence** of firms, as each firm will choose a location knowing that it depends on the one chosen by the other firm.

Assumptions:

- Identical firms are anywhere on a linear market, which has total length 1, where they sell a homogeneous product at an identical price (Bertrand);
- There is a fixed cost of production;
- They want to maximise their share of the market;

- Consumers, uniformly distributed on the line, must buy 1 unit of the product;
- Consumers buy at the nearest firm.

Where should firms locate?

This depends on the number of firms in the market.

- If there is 1 firm:

A monopolist can locate anywhere as people will always have to buy from them.

- If there are 2 firms:

The NE is that they should be in the middle of the line, so that both have a 50% share of the market and neither can improve from this situation.

- If there are 3 firms or more:

This is a tricky question that economic theory still has no definite answer to, so we focus on the case of two firms.

In the **two-case firm**, however, consumers would actually prefer the two firms to be located at $1/4$ and $3/4$ on the line (which reduces the transportation cost of consumers without affecting the firms market shares if both stay put). The problem is that this is not the NE, so firms will not locate this way. In this case, letting market forces decide is not welfare maximising. This could be solved with government intervention. Another problem with this model is that a line is not always the best representation of a market. A better way to represent it would be a circle.

Microeconomics – IBEB – Lecture 18, week 6

Externalities and the Coase theorem

Externalities

There is an externality whenever a decision variable of one economic agent enters directly into the utility function or production function of another. Thus, an **externality** is a *cost* or *benefit* that impacts an agent's **objective function** (utility, profit) even though this agent was neither a buyer nor a seller of the goods/ services causing it.

Negative externality: Involves a *cost*. E.g. pollution

Positive externality: Involves a *benefit*. E.g. a civilised neighbour

Externalities may also be entered indirectly. A famous example is the **pecuniary externality**, which is an externality that enters indirectly through price.

What are the problems of externalities?

Property rights on the externality are not well defined (for example we do not know exactly where air pollution originates from) absence of a market for externalities.

It is difficult to identify who generated the externality. This missing knowledge prevents us from asking or offering compensation.

Examples

1. Pollution clouds from somewhere

In 2017, Russia and Europe found clouds of radioactive Ruthenium-106 in the atmosphere. Still, Russia did not warn Europe about it faster and more precisely, because it was impossible to pinpoint its exact source. This is an example of market failure, as it was not possible to identify the “owner” of the radioactive clouds.

2. Bees and pears

The pear's owner and the bee's owner are neighbours. The pear's owner needs bees to fertilise pear flowers to get pears, and the bee's owner needs flowers where bees can go collect the primary input to produce honey. In this case, both neighbours create *positive externality* for the other. Solution often observed in reality: beekeepers also produce fruits, flowers, etc. so that the externalities are internalised.

Note: This is also observed among Cournot competitors, as each firm creates a negative externality on the profit of its competitor.

3. International Agreements

When an externality has an international dimension, for example CO₂ emissions, it is very difficult to decide who pays for it, who defines the consequences and thus the price, who receives such payment, how to enforce such a system, etc.

4. Education Decision

Let Y be the number of years of education an individual has.

Private profit from studying:

$$BI(Y) = \text{Benefit} - \text{Cost} = 100 * \sqrt{Y} - Y^2$$

Society's profit:

$$BS(Y) = \text{Benefit} - \text{Cost} = (100 + Y) * \sqrt{Y} - Y^2$$

We add Y to 100 because there is a *positive externality* whose benefit is **Y: individual education increases society's human capital**. With optimum for individuals and society are such that $PMB(Y^*) = PMC(Y^*)$ and $SMB(Y^*) = SMC(Y^*)$ respectively, we have that social profit is higher than private profit.

Solution to internalise the externality: **education subsidies**.

5. The tragedy of the commons

"The depletion of a shared resource by individuals, acting independently and rationally according to each one's self interest, despite their understanding that depleting the common source is contrary to their long-term best interests."

This applies to natural resource exploitation, population growth, local government policies financed by the whole country, etc. The problem here is that the negative externality of each individual's behaviour is not internalised by that individual.

Consider a red tuna fisher case.

1. *If they choose the fishing quantity themselves, it would be the NE.*

Problem of each firm:

$$\max \{P(Q)q_i - fq_i\} = [a - b(q_1 + \dots + q_i + \dots + q_n)]q_i - fq_i$$

NE of each firm:

$$q_i^* = \frac{a-f}{(n+1)b}$$

NE of industry: Q^{NE} :

$$\frac{n}{n+1} \frac{a-f}{b}$$

2. *If society chooses the fishing quantity, it would be less than the NE.*

Problem of society:

$$\max \{(a - bQ) Q - fQ\}$$

Equilibrium: $Q^* =$ (when n is large, Q^* is roughly half of Q^{NE})

$$\frac{a-f}{2b} < \frac{n}{n+1} \frac{a-f}{b}$$

Conclusion: What is optimal for each player (NE) is not for society (Pareto); èGet society members to decide jointly or appoint one responsible for choosing the best for society at large.

Externalities and general equilibrium

Competitive equilibrium equalises private MRS and MRT but Pareto optimality equalises social MRS and MRT. In general, the competitive equilibrium is inefficient under externalities and is not a Pareto optimum.

Implications for market structure

With **negative externalities**, having a monopoly may not be as bad as when there are no negative externalities. With **positive externalities**, the presence of a monopoly brings deadweight loss and under-exploitation of positive externality.

Externalities solution and its effect

Suppose firm A produces steel by polluting a river with sulphur dioxide. This imposes a *negative externality* on the fishers downstream. There are two possible solutions:

1. Give fishers the right to have clean water, and the firm needs to pay fishers to let it pollute the river. Problem here is that you may claim your life is put at serious risk just to have the firm pay you as much as possible.
2. Give to firm A the pollution rights, and the fishers can pay the firm to push it to stop polluting.

Although giving property rights does solve the problem at the level of society, the two options have different distributional consequences which may be an issue.

The Coase theorem

If all agents involved in an externality can trade with each other at no cost and any agreement they reach is binding, then the outcome reached is always globally efficient, independently of how the property rights are allocated.

The role of government

Push parties to internalize externality or regulate firms to produce externalities that are the object of the government's attention.

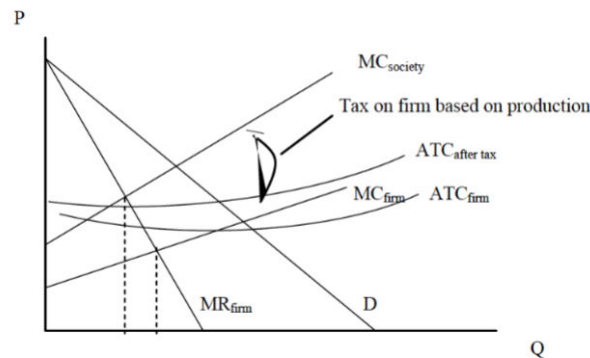
Possible solutions are:

For negative externalities: taxes, quotas (how much you can produce), pollution certificates (firms have to buy certificates to be allowed to pollute) etc.

For positive externalities: subsidies, minimal production requirements, compulsory schooling for kids up to age X, etc.

Issue: the government needs to have a lot of information to intervene at the right level, and government intervention may lead to stark changes in number and characteristics of participants in the market.

The graph below shows the effect of taxation on negative externality done right:



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More on government intervention

Another way to intervene is **to issue licences**, which prove the right of a firm to be active on the market. To decide how many licences to issue, consider the problem of a **circular city**. Suppose:

- The city is a circle of circumference 1 km on which the city needs to locate chemist's shops;
- All shops sell the same drug at the same price;
- There are L customers who are distributed uniformly on the circle;
- Each customer buys 1 unit only of the drug;
- Cost of travel is t euros per km;
- Distance to the shop d is what drives consumer choice;
- As one has to travel to the shop and back home, the cost they have to incur is ;

Each firm's total cost is: (where B is the number of pills.)

$$TC(B) = FC + VC(B) = F + B$$

The cost structure implies that the more firms there are, the costlier the drug is, because $ATC(B) =$

$$\frac{F}{B} + 1$$

is decreasing in B (more firms: walking distances decreases; fewer firms: prices decrease)

The **objective** is to choose the number of firms that minimises the sum of production and transportation costs (maximises production efficiency and customer satisfaction.)

Shops should be located equidistantly on the circle. If there are N shops, the furthest customer is $1/2N$ km away from a shop → on average, a customer is $1/4N$ away from a shop.

Total transportation cost:

$$L * t * 2 * \frac{1}{4N} = \frac{tL}{2N}$$

Total production cost:

$$TC = NF + NB = NF + L$$

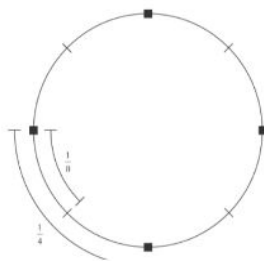
Total cost: TC = transportation cost + production cost =

$$\frac{tL}{2N} + NF + L$$

Minimising total cost, by setting its FOC to 0, yields $N^* =$

$$\sqrt{\frac{tL}{2F}}$$

Thus there will be more chemist shops when t or L increase or F decreases. Vice versa, fewer chemist shops when t or L decreases or F increases.



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