

# EFR summary

Finance 1, FEB12003X

2024-2025



Lectures and Exercise Lectures 1 to 5  
Weeks 1 to 5

**Deloitte.**

DeNederlandscheBank

EUROSYSTEEM

## Details

**Subject:** Finance 1 IBEB 2024-2025

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**Date of publication:** 07.02.2025

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# Finance 1 – IBEB – Lecture 1, week 1

## Types of financial statements

**Balance Sheet.** Firm's financial position at a point in time (firm's assets and liabilities)

**Income Statement.** Firm's earnings (firm's revenues and expenses)

**Statement of Cash Flows.** Indicates the amount of cash generated by the firm.

**Statement of Stockholders' Equity.** Breaks down the stockholders' equity into issuing shares and retained earnings.

## Statement of cash flows

**Free cash flow (FCF)** – the cash flow available for the company to repay creditors, pay dividends and interest to investors.

The statement of cash flow includes **3 sections**: Operating activity, investing activity, and financing activity

Factor	Location
+ EBIT x (1- Tax Rate)	Income Statement
+ Non-cash Expenses (Depreciation, Amortization, etc.)	Income Statement
- Change in (Current Assets - Current Liabilities)	Balance Sheet (current period and previous period)
- Capital Expenditures (CAPEX)	Balance Sheet: Property, Plant, and Equipment (current period and previous period)
= Free Cash Flow	

## Valuation indicators

### Book value versus market value

**Book value:** how accountants evaluate a firm based on the sum of the net profits that were not paid out as dividends over the lifetime of the company. This is the book value of the firm's equity (Equity = Assets - Liability).

Financial economists, on the other hand, assess the value of a firm's equity by looking at its **market value** (or market capitalisation). This equals # shares outstanding \* stock price.

$$\text{market - to - book (MB) ratio} = \text{market value of equity} / \text{book value of equity}$$

When creditors and shareholders have more positive views on the firm's future than the suggestion from its book value, MB is often greater than 1. An obvious example is the potentially huge difference between the market value and the book value of a (great) football player.

**Enterprise value:** the market value of the firm's (underlying) assets that generate cash flows. This is the cost one needs to pay when taking over the enterprise.

$$\text{enterprise value (EV)} = \text{market value of equity} + \text{debt} - \text{cash.}$$

Cash here is the excess cash that is not needed for the firm's operating activities and can be paid back to investors without harming the business. This is different from working capital (cash needed to run the firm).

## Risk-return relations

The higher the risk, the higher the required return.

## No arbitrage

Arbitrage: without taking risks, you make a profit

No arbitrage: you cannot make a profit without taking risks

# Time value of money

Financial decisions are often made by comparing values:

1. Values can be compared only at the same point in time
2. Compound cash flow to move it forward in time

$$FV_n = C * (1 + r)^n$$

3. Discount cash flow to move it backward in time:

$$PV = \frac{C}{(1+r)^n}$$

## Valuing a stream of cash flows

### Present value of a cash flow stream

$$PV = \sum_{n=0}^N \frac{C_n}{(1+r)^n}$$

### Future value of a cash flow stream

$$FV = PV(1 + r)^n$$

**Annuities** - Fixed period. Present value of annuity with growth

$$PV(\text{annuity with growth}) = \frac{C}{(r+g)} \left[ 1 - \left( \frac{1+g}{1+r} \right)^N \right]$$

**Perpetuities** - Infinite life.

$$PV(\text{perpetuity}) = C/r$$

$$PV(\text{growing perpetuity}) = C/(r - g)$$

Growing perpetuity: perpetuity where the payments increase at a constant rate, g.

## Discounting with the risk-free rate

**Risk-free interest rate,  $r_f$** : interest rate at which money can be borrowed or lent without risk over that period

If future payments are risky, premium needs to be added to interest rate to account for riskiness (higher risk => higher interest rate).

# Financial decision-making

Cost-benefit analysis for an investment opportunity can be done by calculating the Net Present Value (NPV) using the formula:

$$NPV = PV(benefits) - PV(costs)$$

NPV decision rule: invest in the alternative with the highest NPV. Choosing this alternative is equivalent to receiving its NPV in cash today.

The net present value of a stream of cash flows can be valued by summing the discounted values of each future cash flow with the appropriate interest rates regarding the time distance.

**Law of one price:** If equivalent investment opportunities trade simultaneously in different markets, then the price of trading should be the same everywhere. If this law does not hold, then an arbitrage opportunity exists.

## Finance 1 – IBEB

### Exercise lecture 1, week 1

## Introduction to financial statement analysis

### Firms' disclosure of financial information

Financial statements are firm-issued accounting reports with past performance information. They are filled with the SEC (Securities and Exchange Commission). Financial statement analysis is used to compare the firm with itself over time, and compare the firm to other similar firms.

### Balance sheet

A firm's balance sheet is a snapshot in time of the firm's financial position.

The balance sheet identity is given by:

$$\text{Assets} = \text{Liabilities} + \text{Stockholders' Equity.}$$

**Assets:** what the firm owns.

- Current assets: cash or assets that are expected to be turned into cash within a year. This category includes cash, marketable securities (short-term low-risk investments like government bonds), accounts receivable, inventories, and other current assets such as pre-paid expenses.
- Long-term assets include net property, plant, and equipment (book value = cost of acquisition - accumulated depreciation), goodwill and intangible assets and other long-term assets, such as investments in long-term securities.

**Liabilities:** what the firm owes.

- Current liabilities are to be paid within a year. This includes accounts payable, short-term debt/notes payable, current maturities of long-term debt, and other current liabilities such as taxes payable, wages payable.

\***Net working capital** is the capital that is available in the short term to run the business:

$$\text{Net working capital} = \text{current assets} - \text{current liabilities.}$$

- Long-term liabilities consist of other liabilities with the maturity of longer than one year and include long-term debt, capital leases, and deferred taxes.

**Stockholders' Equity:** the difference between the value of the firm's assets and liabilities.

- Book value of equity can be negative because it is calculated as the difference between book value of assets and book value of liabilities. However, many of the firm's valuable assets may not be reflected in the balance sheet (for example: the firm's reputation).

- Market value of equity (Market Capitalization) = Market price per share × number of shares outstanding. This cannot be negative and often differs substantially from book value.

- Market-to-book ratio (or Price-to-book ratio)

$$\text{market-to-book (MB) ratio} = \text{market value of equity} / \text{book value of equity}$$

Value stocks:  $MB \text{ ratio} < 1$

Growth stocks:  $MB \text{ ratio} > 1$

- Total enterprise value (TEV)

$$\text{enterprise value (EV)} = \text{market value of equity} + \text{debt} - \text{cash}$$

## Income statement

Income statement indicates the flows of revenues and expenses over a period of time.

- An important component of an income statement is the “bottom line” (net income = earnings in a period).

### Earning calculations:

Total sales/revenue

-

Cost of sales

-----

Gross profit

-

Operating expenses

-----

Operating income

+/-

Other income/expenses

-----

Earnings before interest and tax (EBIT)

+/-

Interest income/interest expenses

-----

Pre tax income

-

Taxes

-----

Net income

Net income/No of shares outstanding = EPS



## Statement of Cash Flows

Net income typically does not equal the amount of cash the firm has earned, because it includes non-cash expenses such as depreciation and amortization, and excludes cash uses such as investment in property, plant, and equipment or expenditures on inventory.

A statement of cash flows can be used to calculate free cash flows (FCF) and enterprise value. It includes three sections:

**1. Operating Activity:** Adjusts net income for all non-cash items related to operating activities and changes in net working capital.

Adjustments:

- Depreciation / amortization: add the amount of depreciation / amortization (as a non-cash expense)
- Account receivable: deduct the increases (as the cash is not yet been received)
- Accounts payable: add the increases (cash have not been paid yet)
- Inventories: deduct the increases (any increases in inventory are paid by cash)

**2. Investment Activity:** all cash required for investment activities

- Capital expenditures (purchasing PPE)
- Trading of marketable securities
- Acquisition related expenditures

**3. Financing Activity:**

- Payments of dividends (cash outflow, therefore is deducted)  
**retained earnings** = net income - dividends
- Changes in borrowings (increases in borrowings are cash inflows)

## Financial decision-making and law of one price

**Financial decision making:** Investment should be made when

$PV(\text{benefits}) > PV(\text{costs})$ .

$$NPV = PV(\text{benefits}) - PV(\text{costs}) \rightarrow NPV > 0$$

**Arbitrage** refers to taking advantage of the price difference when buying and selling equivalent goods in different markets. An arbitrage opportunity occurs when it is possible to make a profit without taking any risk or making any investment.

**Normal market** is a competitive market in which there is no arbitrage opportunity.

**Law of One Price:** If equivalent investment opportunities are traded at the same time in different normal markets, then they must trade for the same price in both markets.

## Time value of money

### PV shortcut formulas

#### Perpetuities

- Constant cash flow:  $PV = \frac{C}{r}$
- Growing cash flow:  $PV = \frac{C}{(r-g)}$

#### Annuities

- Constant cash flow:  $PV = \frac{C}{r} \times \left[ 1 - \frac{1}{(1+r)^n} \right]$
  - Growing cash flow:  $PV = \frac{C}{r-g} \times \left[ 1 - \left( \frac{(1+g)}{(1+r)} \right)^n \right]$
- (g=growth rate; n=number of periods; C=cash flow; r=interest rate)

Type of cash flows	Constant cash flows	Growing cash flows
<b>Perpetuities</b> (last forever)	$g = 0, n \rightarrow \infty$ $PV (\text{perpetuity}) = \frac{C}{r}$	$g < r, n \rightarrow \infty$ $PV (\text{growing perpetuity}) = \frac{C}{(r-g)}$
<b>Annuities</b> (N periods)	$g = 0, n \rightarrow N$ $PV (\text{annuity}) = \frac{C}{r} \times \left[ 1 - \frac{1}{(1+r)^N} \right]$	$n \rightarrow N$ $PV (\text{growing annuity}) = \frac{C}{r-g} \times \left[ 1 - \left( \frac{(1+g)}{(1+r)} \right)^N \right]$

# Interest rates

**Effective Annual Rate (EAR)** indicates the total amount of interest that will be earned at the end of one year. Typically used in present value calculations for yearly cash flows as it considers the effect of compounding. Also referred to as the Effective Annual Yield (EAY) or Annual Percentage Yield (APY).

**Annual Percentage Rate (APR)** indicates the amount of simple interest earned in one year.

**Simple interest** is the amount of interest earned without the effect of compounding. The APR is typically less than the EAR.

\* Note that the APR cannot be used as a discount rate without adjustments made.

The APR with k compounding periods is a way of quoting the actual interest earned each compounding period:

$$\text{Interest Rate per Compounding Period} = APR / \left( \frac{k \text{ periods}}{\text{year}} \right)$$

To convert an APR to an EAR, we can use the following formula:

$$1 + EAR = \left( 1 + \frac{APR}{k} \right)^k$$

## Finance 1 – IBEB

## Lecture & Exercise lecture 2, week 2

### Valuing bonds

**Bond:** A tradable loan, which generally has a fixed maturity and fixed coupon payments (cash flow stream).

#### 2 types of bonds

**1. Zero coupon bond:** Offers a single payment.

- Purchaser pays the price of the bond
- The bond promises in K-year to pay the bondholder a single payment, called face value (par value)
- The date of this payment is called the maturity date. K is the time to maturity

**2. Coupon bond:** Zero coupon bonds with additional periodic payments

- The bond promises in K-year to pay the bondholder a single payment, called face value
- The date of this payment is called the maturity date. K is the time to maturity
- Additional coupons: At regular intervals until maturity, the bondholder receives a coupon payment
- Coupon rate: the ratio of total annual coupon to face value

## Bond price: present value of cash flows

**Step 1:** Identify all cash flows

**Step 2:** Discount these cash flows using rules of time travel. For very long-term bonds, we can use the annuity formula for the coupons + present value of final payment of face value

- Annuity without growth

$$PV_A = \frac{C}{r} \left[ 1 - \left( \frac{1}{1+r} \right)^T \right]$$

- Bond value = PV (coupon value) + PV (face value)

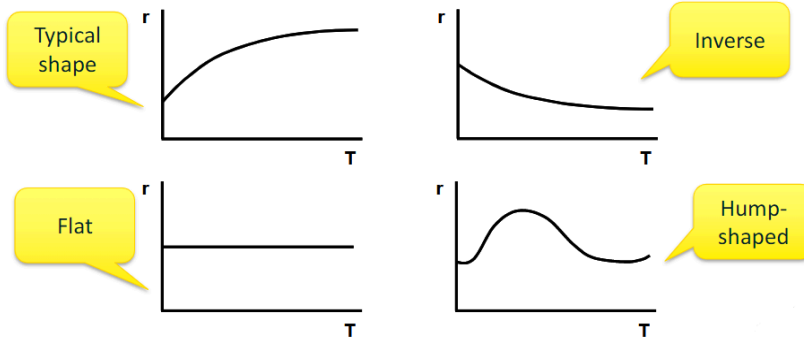
$$PV_B = \frac{Coupon}{r} \left[ 1 - \left( \frac{1}{1+r} \right)^T \right] + \frac{Face\ value}{(1+r)^T}$$

## What drives bond prices

### 1. Interest rate

When interest rates rise, bond prices typically decline.

**Term structure** of interest rates indicates how interest rates vary with the maturity at one point in time. The most typical case depicts an upward-sloping term structure, because the longer the time until maturity the higher the interest rate.



## 2. Risk of default

The possibility of default adds a credit spread, increasing the required return and lowering the bond's price.

$$Price = \sum_{t=1}^T \frac{Face\ value}{(1+r_f+credit\ spread)^t}$$

## Bond price: yield to maturity

The bond **yield** is another way to express the bond price:

$$Price = \sum_{t=1}^T \frac{C_t}{(1+r_t)^t} = \sum_{t=1}^T \frac{C_t}{(1+YTM)^t}$$

\*YTM: weighted average of spot rate used for discounting.

## Spot rates and forward rates

The difference between the spot rate (r) and forward rate (f) is that the spot rate always starts at time t=0 and lasts until a further point in time regardless how far this point is. But forward rates last from one point in time to the next point in time.

The law of one price must hold, therefore the general formula for period n:

$$(1 + f_{n-1,n}) = (1 + r_n)^n / (1 + r_{n-1})^{n-1}$$

# Investment decision rules

## 1. Net present value (NPV)

$$NPV = \sum_{t=0}^T CF_t / (1 + R_t)^t = -INV_0 + \sum_{t=1}^T CF_t / (1 + R_t)^t$$

Choose the investment with the highest NPV

### Complexities in Using NPV

1. Comparing projects with varying durations.
2. Capital constraints

### Duration of the projects

When choosing from the projects with different duration, we have to extend the timeline of the projects as fair comparison can be done only if periods are equal.

### Capital constraints

Sometimes firms have many positive NPV projects and they can't invest in all of them simultaneously

To tackle this, we take the following steps:

Step 1: Form all possible combinations between available projects

Step 2: Cancel those combinations that are impossible due to limited budget

Step 3: Choose the combination with the highest NPV

**Profitability index** (NPV per constrained resource) = NPV/Investments

It can be used to identify the optimal combination of projects to invest in under the budget constraint. Rank projects by PI and select the ones with the highest indicator.

$$PI = \frac{\text{value created}}{\text{resource consumed}} = \frac{NPV}{\text{resource consumed}}$$

Strengths of using NPV

- Often right and unambiguous
- Incorporates time value of money
- Easy to compare between projects
- Takes investment size into account
- Can compare one large project versus several smaller ones

Weakness:

- Needs an appropriate discount rate

## 2. Payback period

**The payback period (PBP):** The amount of time it takes to recover or pay back the initial investment

=> If the payback period is less than a pre-specified length of time, you should accept the project. Otherwise, you reject it

**Formula:**

$$INV_{t=0} \leq \sum_{t=1}^{PBP} CF_t$$

Pitfalls of the PBP method

- Ignores the project's capital cost and value of money
- Ignores cash flows after the payback period

Even though the answer is not always correct, this alternative method is used because of uncertainty in the long run, because the future is hard to predict.

## 3. Internal rate of return (IRR)

**Internal rate of return (IRR)** is the interest rate at which the NPV of an investment project equals zero.

- Invest if  $IRR > \text{capital cost}$  and don't invest if  $IRR < \text{capital cost}$
- If  $IRR > r$  this indicates that  $NPV > 0$  and hence it is attractive to do the project.
- Choose the project with the highest IRR

The IRR can only be deduced by trial and error and the formula is such that:

$$0 = \sum_{t=0}^T CF_t / (1 + IRR)^t$$

Pitfalls of the IRR method: It ignores the size of a project, and with longer duration projects where reinvestment is assumed this might be a disadvantage. In case of

positive and negative cash flows there is a possibility for multiple IRRs to exist with positive and negative interest rates when NPV is zero.

## Capital budgeting

The process of analyzing investment opportunities and deciding which one to choose is called capital budgeting.

**Incremental** cash flows = the difference between doing and not doing the project.

**Cannibalization** occurs when the introduction of a new product has an adverse impact on the sales of existing products.

**Opportunity cost** - the revenue that could have been earned with an alternative use of the asset.

**Sunk costs** - unrecoverable costs that are already incurred and therefore irrelevant for the decision making.

$$\text{net working capital} = \text{current assets} - \text{current liabilities}$$

$$\text{Free Cash Flow} = (\text{Revenues} - \text{Costs} - \text{Depreciation}) \times (1 - \tau) + \text{Depreciation} - \Delta\text{NWC} - \text{CapEx}$$

## Dealing with uncertainty

- **Break-even** analysis: this analysis finds the value of a parameter for which the  $NPV = 0$ . After this we can determine how likely it is that the parameter value is below or above the break-even amount
- **Sensitivity** analysis: input assumptions in this analysis will be changed within a plausible range, which will affect the NPV.
- **Scenario** analysis: in this analysis, the values of parameters will be determined for multiple scenarios, like a best case and worst case. These scenarios will be compared by their NPV when multiple parameters are changed at the same time.



# Valuing stocks

For bonds, the cash flows were known, they consisted of coupons and a principal. But stock only pays dividends, we get the value of what is left of the earnings of a company, which are not known in advance.

## Terminology

- Common Stock: security representing a share in the ownership of a corporation.
- Initial Public Offering: the first sale of stock in a corporation to the public.
- Secondary Market: a market, often a stock exchange, in which previously issued shares are traded amongst investors.
- Dividends: payments made by companies to shareholders.
- Dividend yield: ratio of annual dividend to share price. There are different types (for example, stock or cash; preferred and common)
- P/E Ratio: share price divided by earnings per share (price-to-earnings)

Two ways to estimate the value of a stock: **Dividend Discount Model (DDM)** or estimating the **value using comparable firms**.

## Dividend Discount Model (DDM)

$$R_E = R_t + \text{equity risk premium}; P_0 = \frac{div_1}{1+r_E} + \frac{P_1}{1+r_E} \Rightarrow r_E = \frac{div_1}{P_0} + \frac{P_1 - P_0}{P_0}$$

where  $\frac{div_1}{P_0}$  represents dividend yield, and  $\frac{P_1 - P_0}{P_0}$  the capital gains rate. If we assume there will be no dividend growth, we use the following formula to calculate the value of a stock:

$$P_0 = \sum_{t=1}^{\infty} div / (1 + R_E)^t$$

The **Gordon growth model** assumes a constant dividend growth,  $g$ , shown in the formula:

$$P_0 = \sum_{t=1}^{\infty} (div \times (1 + g)^{t-1}) / (1 + R_E)^t = \frac{div}{R_E - g}$$

Disadvantages of this model: Everything is very dependent on the first dividend forecast, if this is wrong, this has a big impact on the value of the stock. Also, the growth rate cannot be bigger than the equity cost of capital here.

## Multiple valuation of comparable stocks

A company does not have to pay all earnings, they can keep them as retained earnings, which they can use for new investments.

$$div_t = EarningsPerShare_t \times Dividend\ payout\ rate\ (k)_t$$

$$EPS = \frac{earnings_t}{shares\ outstanding}$$

By combining this formula with dividend discount model we can arrive at P/E ratio:

$$\frac{Price}{Earnings} = \frac{k}{R_E - g}$$

# Finance 1 – IBEB – Lecture and Exercise lecture 3, week 3

## Valuing stocks

### Information

The first version of the three levels of informational efficiency was constructed by Fama in 1970:

- Weak: The stock prices reflect all information on its historical price. In technical analysis, trends of history are used to predict the future, but these cannot be considered useful.
- Semi-strong: the prices of the stock are based on the publicly available information which is relevant for pricing. The prices adjust to public information.
- Strong: not only is the publicly available information reflected in the prices, but also insider information of firms which is not yet public to all investors.

### Lessons from efficient markets

**Efficient market hypothesis** - when relevant information is available it is immediately and completely reflected in prices, therefore it is not possible to systematically achieve abnormal profits.

For investors to have positive-NPV trade opportunities, competitive advantage is needed, with low transaction costs, no regulation, no benchmark index, etc. In this scenario, research is not needed and investors can just buy stock at or close to fair value.

Corporate managers should focus on finding positive-NPV projects in real markets, they will be fairly financed. Analysts will focus on free cash flow to determine the value.

In efficient markets, no arbitrage opportunities can exist. The same amount of risk corresponds to the same expected return. To determine whether a market is efficient, we need a theory for risk and return.

## Capital markets and the pricing of risk

### Risk and return measures

#### Expected return

$$E[R] = \mu = \sum_{\text{all possible } R} \text{probability}_R \times R$$

**Variance** - indicates how much the squared deviation from the mean is, which can be calculated with the formula:

$$\text{variance} = E[(R - E[R])^2] = \sum_{\text{all possible } R} \text{probability}_R \times (R - E[R])^2$$

The standard deviation,  $\sigma(R)$ , which is the square root of the variance, in Finance is also known as **volatility**. It measures how returns vary with the spread of the distribution of the return. The bigger the volatility, the higher the risk, which is compensated with a higher return..

$$\text{volatility} = \text{standard deviation} = \sigma = \sqrt{\text{variance}}$$

#### Covariance

$$\text{cov}_{AB} = \sum_{j=1}^z p_j \times [R_{A,j} - E(R_A)] \times [R_{B,j} - E(R_B)]$$

## Historical returns

Instead of using expected outcomes, historical data can also be used to calculate variance. However, they are often uncertain with problematic data

Formula when estimating using sample of historical returns

**Expected return:**  $E(R_A) = \bar{R}_A = \frac{1}{N} \sum_{t=1}^N R_{A,t}$

**Variance:**  $\sigma_A^2 = \frac{1}{N-1} \sum_{t=1}^N [R_{A,t} - \bar{R}_A]^2$

**Covariance:**  $cov_{AB} = \frac{1}{N-1} \sum_{t=1}^N [R_{A,t} - \bar{R}_A] \times [R_{B,t} - \bar{R}_B]$

Positive linear correlation  $cov_{AB} > 0$

Negative linear correlation  $cov_{AB} < 0$

**Correlation coefficient:**  $\rho_{AB} = \frac{cov_{AB}}{\sigma_A \sigma_B}$

## Diversification in stock portfolios

There will be no risk premium for diversifiable (=idiosyncratic) risk, thus investors will not be compensated for holding firm-specific risk. This is because if they diversify their portfolio, this risk will be eliminated for free.

A risk premium will only be determined by the systematic risk, which cannot be avoided even when a portfolio is diversified.

$$volatility = total\ risk = systematic\ risk + diversifiable\ risk$$

What should determine the expected return of an asset

$\beta$  - the sensitivity of the asset to movements of the market portfolio;

The expected percentage change in the excess return of an asset for a 1% change in the excess return of the market portfolio. (Beta is a measure of systematic risk)

$$\beta_i = \frac{cov(R_i, R_{mkt})}{var(R_{mkt})} = \frac{\sigma(R_i) \times \rho(R_i, R_{mkt})}{\sigma(R_{mkt})}$$

$$E[R_i] = \text{risk free rate} + \text{risk premium} = R_f + \beta_i \times (E[R_m] - R_f)$$

## Optimal portfolio choice

The portfolio weight is the fraction of a stock in the total portfolio held by an individual. Thus, if we have n different investments, we call this an n-stock portfolio.

$$\text{portfolio weight } (w_i) = \frac{\text{value of asset } i}{\text{total value of portfolio}}; \sum_{i=1}^N w_i = 1$$

Also, the expected return of a portfolio will be equal to the sum of the expected returns of all investments held in the portfolio.

$$E[R_p] = \sum_{i=1}^N w_i \times E[R_i]$$

And the variance of the return of this portfolio can be calculated with:

$$\text{Var}[R_p] = \sum_{i=1}^N \sum_{j=1}^N w_i \times w_j \times \text{Cov}\{R_i, R_j\}$$

When choosing an efficient portfolio, we want to choose the lowest risk given a certain level of expected return, or if the risk is given, we will want to maximise the expected return.

## Finding minimum variance portfolio

$$\min (w_1, w_2) w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \sigma_{12} \quad ; w_1 + w_2 = 1$$

$$\Rightarrow \min (w_1) w_1^2 \sigma_1^2 + (1 - w_1)^2 \sigma_2^2 + 2w_1(1 - w_1) \sigma_{12} \quad ; \partial/\partial w_1 = 0$$

$$\Rightarrow w_1 = \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}$$

## Capital Asset Pricing Model (CAPM)

CAPM implies that investors hold a portfolio that consists of two assets:

1. Market portfolio
2. Risk-free investment

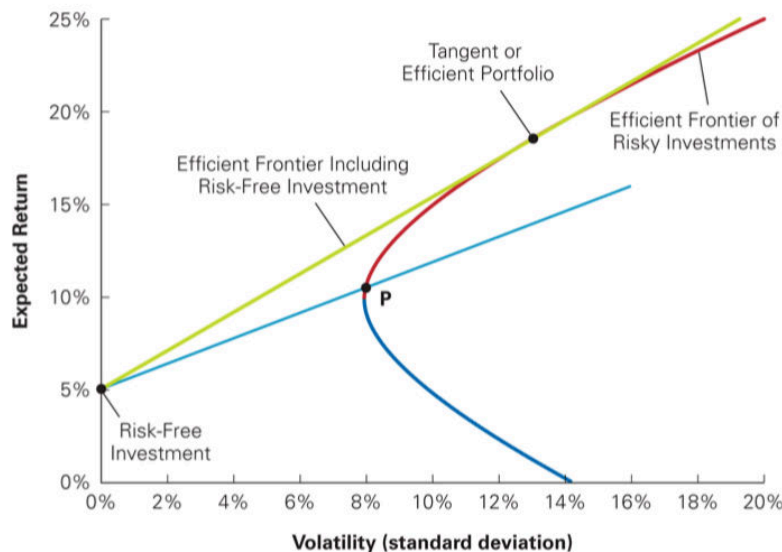
## The CAPM equation

$$E(R_i) = R_f + \beta_i(E(R_M) - R_f), \quad \beta_i = \frac{\text{Cov}(R_i, R_M)}{\text{Var}(R_M)}$$

## Capital market line

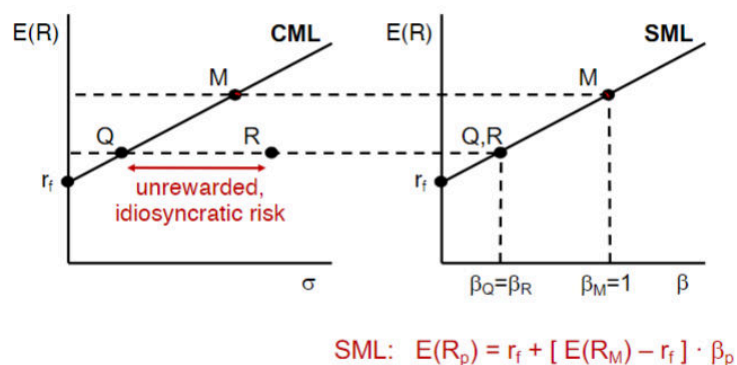
**The capital market line (CML)** is the best expected return that can be obtained for each level of the total risk (volatility). The CML is in fact the efficient frontier.

All the stocks and portfolios that are to the right of the CML have some diversifiable risk, on the CML we only have systematic risk.



## Security market line

The security market line (SML) goes through the points of risk-free investment and market portfolio. If there will be a negative beta for a security, this means the security does well when the market is not doing well. The expected return of the security will be lower than that of  $R_f$ , the risk premium is smaller than zero, but this does not mean it is risk free. This security can be held with other securities to diversify the portfolio, but this does decrease the return.



The SML matches the expected return for each security with the corresponding beta to the market. Because all stocks and portfolios are part of the efficient market portfolio, they should lie on the SML.

### Beta( $\beta$ ) of a Portfolio

The beta of the portfolio corresponds to the weighted average beta of the securities in the portfolio and can be calculated using the formula:

$$\beta_p = \frac{\text{cov}\left(\sum_i w_i R_i, R_{mkt}\right)}{\text{var}(R_{mkt})} = \sum_i w_i \frac{\text{cov}(R_i, R_{mkt})}{\text{var}(R_{mkt})} = \sum_i w_i \beta_i$$

# Finance 1 – IBEB – Lecture 4 and Exercise lecture 4, week 4

## Cost of capital

Resources are not obtained by a firm for free, because this amount could have been used to find other projects. Thus, the opportunity cost of using this resource is equal to the value it would have when using the resource for the best possible alternative. The cost of capital of an investment is the expected return of available investments with the same beta.

- Debt cost of capital
- Equity cost of capital

- Project cost of capital

## Equity cost of capital

The CAPM can be used to estimate cost of capital by the Security Market Line (SML) equation:  $r_i = r_f + \beta_i \times (E(R_{mkt}) - r_f)$

### The Market Portfolio

**Value-Weighted Portfolio** - a portfolio that consists of securities that are held in proportion to their market capitalization.

A Value-weighted portfolio is also an **Equal-Ownership portfolio** as it contains an equal fraction of the total number of shares outstanding of each security in the portfolio.

Additionally, it is also a **Passive portfolio** that does not require rebalancing even if market prices change.

In a portfolio like the market portfolio the investment in each security  $i$  is proportional to its market capitalization:

$$MV_i = \# \text{ Shares outstanding} \times \text{ Price of } i \text{ per share}$$

The weight of each security in the portfolio can be calculated as follows:

$$x_i = \frac{\text{Market Value of } i}{\text{Total Market Value of all securities}} = MV_i / \sum_j MV_j$$

### The Market Risk Premium

The risk-free rate is most commonly determined by the yield on US Treasury securities.

Market risk premium can be approximated by so-called **market indices**. For example, S&P 500 (value-weighted portfolio).



The expected return on the market can be determined using the historical average of market return. However, there is a drawback for using historical data. It is backward-looking and, hence, may not be representative of expectations about the future.

A possible alternative could be to look at what is implied by the market and solve for the discount rate:  $r_{mkt} = \frac{div_1}{P_0} + g = \text{Dividend yield} + \text{Expected dividend growth rate}$

## Beta estimation

Linear regression can be applied to estimate the excess return on stock i (dependent variable) with market excess return (independent variable):

$$(R_i - r_f) = \alpha_i + \beta_i(R_{mkt} - r_f) + \varepsilon_i$$

Where  $\alpha_i$  is the intercept term,  $\beta_i(R_{mkt} - r_f)$  is the sensitivity of the stock to market risk and  $\varepsilon_i$  is the error term (zero on average)

Essentially,  $\alpha_i$  is the measure of historical return on stock against the estimation of SML.

- Positive alpha means that historical return on stock was better than predicted by CAPM
- Negative alpha means that historical return was below SML.

## Debt cost of capital

**Yield to maturity** is the IRR an investor will earn from holding the bond to maturity and receiving its promised payments.

Yield to maturity is a reasonable estimation of investors' expected rate of return if there is little risk of default. If there is a high risk of default, yield to maturity exceeds investors' expected return.

Expected return of the bonds can be calculated using the formula:

$$\begin{aligned} r_d &= (1 - p)y + p(y - L) = y - pL = \\ &= \text{Yield to Maturity} - \text{probability of default} \times \text{Expected loss rate} \end{aligned}$$

# Project's cost of capital

## All-equity comparables

To measure the compensation for the risk of a new project when it is fully equity financed, but different from the average project in the company, we want to find an all-equity financed firm with similar business operations to a new project for comparison.

By using comparable firm's beta and cost of capital we can estimate the cost of capital for the new project. If it is not possible to find an all-equity financed firm we can also make estimates from a levered firm (financed by both debt and equity) with similar business activities. As a result of having debt financing the return on equity will be higher due to higher risk. Hence we want to use the unlevered cost of capital.

The **unlevered cost of capital** - the expected return required by investors to hold the firm's underlying

The weighted average of the firm's equity and debt cost of capital can be calculated using the formula:

$$r_u = r_a = \frac{E}{E+D} \times r_E + \frac{D}{E+D} \times r_D$$

where  $r_a$  stands for return on assets

\*note: "perfect" world without taxes and frictions assumed

Similarly:

$$\beta_u = \frac{E}{E+D} \times \beta_E + \frac{D}{E+D} \times \beta_D$$

When calculating  $\frac{D}{E+D}$  ratio it is important to use the net debt. This is because cash is a risk-free asset that reduces the average risk of a firm's assets.

*Net Debt = Debt - Excess cash - Short term investments*

**Operating Leverage** is the proportion of fixed to variable costs of the project. A higher proportion of fixed costs implies higher sensitivity of the project's cash flows to market risks, meaning higher beta and cost of capital.

## The Weighted Average Cost of Capital (WACC)

When the assumption of the “perfect” world is relaxed, we need to account for taxes.

$$\text{Effective after-tax interest rate} = r(1 - \tau_c)$$

The Weighted Average Cost of Capital is then equal to:

$$r_{WACC} = \frac{E}{E+D} \times r_E + \frac{D}{E+D} \times r_D \times (1 - \tau_c)$$

Given a target leverage ratio:

$$r_{WACC} = r_u - \frac{D}{E+D} \times \tau_c r_D$$

Where  $r_u$  is the unlevered cost of capital or pre-tax WACC.

### WACC and pre-tax WACC

Unlevered cost of capital shows the expected return on holding a firm's assets, which in the real world with taxes can be used for evaluation of all-equity projects with the same risk as the firm.

Taking in account taxes, WACC adjusts for the capital structure of the firm.

## Investor behavior and capital efficiency

### Competition and Capital markets

**Stock's alpha** - the difference between a stock's expected return and its required return according to the security market line (CAPM return). When the market portfolio is efficient all stocks are on SML. If there is no deviation from SML, alpha is equal to zero.

$$\text{Stock's alpha: } \alpha_s = E[R_s] - r_s$$

$$\text{Security market line: } r_s = r_f + \beta_s \times (E[R_{mkt}] - r_f)$$

## Deviation from SML

Stock prices adjust to news

- Positive alpha (=higher expected return)
- Investors buy at a lower price than CAPM prediction until expected returns go down again
- The stock is back on SML

## Information and rational expectation

In CAPM investors should hold a combination of market portfolio and a risk-free investment.

The market portfolio can be inefficient if

- Information was misinterpreted by a substantial number of investors;
- A significant number of investors are willingly hold inefficient portfolio because they also care about other aspects of portfolios other than returns and volatility

## Systematic trading biases

Biases become problematic when they occur systematically instead of randomly.

Some common biases are:

- **Disposition effect**- tendency to hold losing stocks for too long, as investors do not want to realize losses
- **Herd behavior**- occurs when individuals start actively following each other

## Multiple factor model

CAPM used so far is an example of a single-factor model.

**Multi-factor model of risk** is used where there are other factors that explain returns, hence, CAPM proposed portfolios may not be efficient. In this case, to construct efficient portfolios we need to use factor portfolios that are covering different risk factors.

Multifactor model of risk for given N factor portfolios:

$$\begin{aligned}
E[R_s] &= r_f + \beta_s^{F1} \times (E[R_{F1}] - r_f) + \beta_s^{F2} \times (E[R_{F2}] - r_f) + \dots + \beta_s^{FN} \times (E[R_{FN}] - r_f) \\
&= r_f + \sum_{n=1}^N \beta_s^{FN} \times (E[R_{FN}] - r_f)
\end{aligned}$$

## Self-financing portfolio

A self-financing portfolio can be constructed by going short in some stocks and long in others with the same market value.

For self-financing factor portfolios the formula comes down to (as the risk free rate drops out):

$$\begin{aligned}
E[R_s] &= r_f + \beta_s^{F1} \times E[R_{F1}] + \beta_s^{F2} \times E[R_{F2}] + \dots + \beta_s^{FN} \times E[R_{Fn}] = \\
&= r_f + \sum_{n=1}^N \beta_s^{FN} \times (E[R_{FN}])
\end{aligned}$$

## Arbitrage Pricing Theory (APT)

There are no systematic possibilities available to investors for arbitrage, therefore the arbitrage pricing relationship (APR) holds:

$$E[R_i] = \lambda_0 + \beta_{i,1}\lambda_1 + \beta_{i,2}\lambda_2 + \dots + \beta_{i,k}\lambda_k$$

Where  $\lambda_0$  corresponds to risk-free rate, other  $\lambda$  are risk premiums for different factors that together measure the systematic risk,  $\beta$  is the sensitivity of the security to each risk factor.

# Finance 1 – IBEB – Lecture and Exercise lecture 5, week 5

## Capital structure in a perfect market

The **capital structure** of a firm indicates whether the firm's assets are financed with equity or debt. An **unlevered firm** will only use their own resources to finance projects. A **levered firm** instead used debt and sometimes equity to finance the projects.

### Modigliani-Miller

Notation: E- market value of equity in a levered firm, D- market value of debt in a levered firm, U- market value of equity in an unlevered firm, A- market value of firm's assets.

**Proposition I:** In the presence of a perfect capital market, the cash flows generated by a firm's assets equals the total value of the firm, and this cannot be affected by the chosen capital structure of the firm.

$$E + D = U = A$$

Changes in capital structure only affect the distribution of the cash flows among stakeholders, but not the cash flows themselves. Financing with more debt will make equity more risky, so higher equity cost of capital will offset the low debt cost of capital.

As previously discussed, the return on unlevered equity is related to the returns of levered equity and debt:  $R_u = R_a = \frac{E}{E+D} \times R_E + \frac{D}{E+D} \times R_D$

Solving for  $R_E$ :  $R_E = R_u + \frac{D}{E}(R_u - R_D)$

**Proposition II:** Following from rewritten formula, the cost of capital of levered equity is equal to the cost of capital for unlevered equity plus an additional risk premium due to leverage, that is proportional to the firm's market value debt-to-equity ratio.

**Beta:** Similarly, the effect of leverage on the risk of security can be expressed in terms of beta:  $\beta_E = \beta_u + \frac{D}{E}(\beta_u - \beta_D)$

## Equity Issuances

**Dilution**- an increase in the total number of shares that will divide a fixed amount of earnings.

If a firm issues new shares, there will be no gain or loss for the shareholders if the shares are sold at a fair price. This is because the price of the share has to remain constant. But we do have **earnings dilution** in this case, this is because the same amount of earnings now has to be divided among more shares. Which leads us to the **dilution fallacy**, that if a firm starts issuing equity, this will reduce the value of the existing shares of the shareholders. But here it is not taken into account that if a firm starts issuing new shares, the cash earned by a firm will increase, thus a firm will have more assets and higher cash flows. The increases of this will be equal to the dilution of the shares. The fallacy should not affect the decision on equity vs debt financing.

## Capital structure with corporate tax

Until now we assumed that the capital structure a firm chooses does not influence the value of the firm in a perfect capital market. But in fact, the amount of leverage a firm has does affect the value of the firm via the corporate tax. Firms have to pay tax on their EBIT. The interest expense is the amount of payments made to debt holders, and the left-over cash flows that are paid to debt holders. The income before taxes are deducted is the pre-tax income. The corporate taxes will be the corporate tax rate,  $\tau_c$  times the earnings of a firm minus the interest payments. The net income a firm will have left is the cash flow which can be used to pay equity holders,  $(1 - \tau_c)(EBIT - r_D D)$ . As the interest expense reduces the amount of corporate tax, it incentivises the use of debt. The reduction in taxes paid due to interest expenses can be calculated as follows:

$$\text{Interest Tax Shield} = \text{Corporate Tax Rate} \times \text{Interest payments}$$

## Modigliani-Miller with taxes

**Proposition I:** the value of firms with leverage will be higher than that without leverage due to tax savings from the debt.

$$V^L = V^U + PV(\text{Interest Tax Shield})$$

If the debt a firm has issued comes due, it will be refinanced with new debt. This way the debt is permanent, and they use the interest tax shield as a perpetuity.

$$PV(\text{Interest Tax Shield}) = \tau_c D$$

### Adjusting WACC

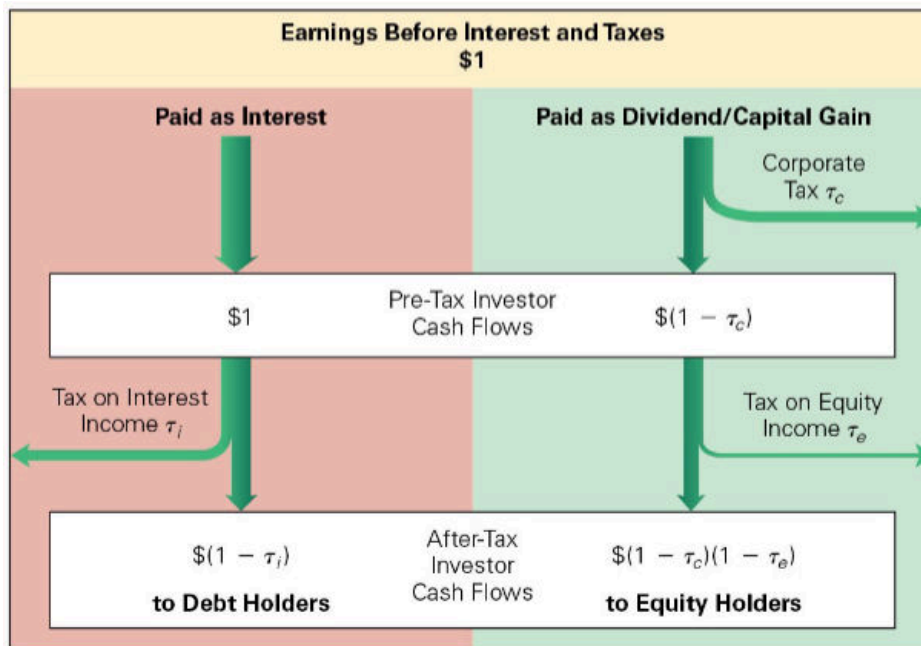
The weighted average cost of capital (WACC) (including taxes) will give us the effective after-tax cost of capital to the firm, where the reduction of the interest tax shield is taken into account. The Weighted Average Cost of Capital is then equal to:

$$r_{WACC} = \frac{E}{E+D} \times r_E + \frac{D}{E+D} \times r_D \times (1 - \tau_c)$$

## Capital structure with corporate tax and personal taxes

Notation:  $\tau_c$  - corporate tax rate,  $\tau_i$  - personal tax rate on interest payments (debt income),  $\tau_e$  - personal tax rate on dividends and capital gains (income from equity)





Lecture 5, dr. Matthijs Korevaar (2024)

Now we assume that individuals also pay taxes, debt holders pay tax on interest payments, which is typically taxed as income, and equity holders pay taxes on dividends. The **effective tax advantage of debt** (with personal tax) can be computed as follows:

$$\tau^* = \frac{(1-\tau_i) - (1-\tau_c)(1-\tau_e)}{(1-\tau_i)} = 1 - \frac{(1-\tau_c)(1-\tau_e)}{(1-\tau_i)}$$

With personal taxes, the firm's equity and debt cost of capital will adjust to compensate investors for their tax burdens, therefore, WACC will be calculated using the same formula as before:

$$r_{WACC} = \frac{E}{E+D} \times r_E + \frac{D}{E+D} \times r_D \times (1 - \tau_c)$$

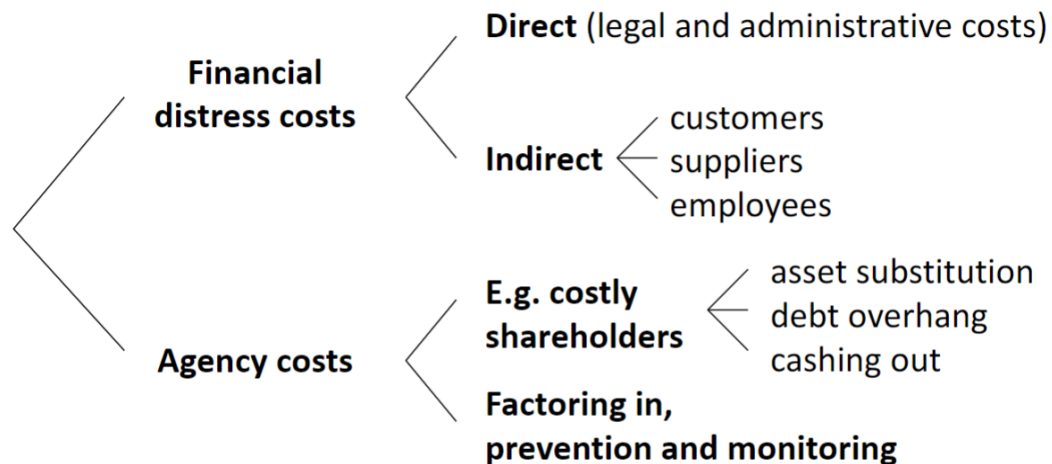
## Financial distress, managerial incentives & information

**Financial distress:** When a firm has trouble with paying the debt obligations that are due as a result of unstable cash flows.

A firm **defaults** if it fails to make any of these payments to debt holders.

Leverage increases the risk of bankruptcy as debt is a formal commitment to pay the debt holder, whereas in equity financing a firm is not obligated to pay dividends.

Debt holders will receive certain rights to the assets of the firm when a firm defaults. A **bankruptcy** shifts ownership of the firm from equity holders to debt holders. This does not have any effect on the total value that is available to all investors.



The cost of capital of debt will be higher than the risk-free rate. When debt is low and leverage increases, the cost of capital for equity holders will be higher because risk is shared among more equity holders. But this is compensated by higher returns. But as debt keeps increasing the cost of capital for debt will start to rise as well, which is the same for the cost of equity but less strong.

The **costs of bankruptcy** can be divided among two groups, the direct and indirect costs.

- **Direct costs:** legal costs (decrease the firm's asset values).
- **Indirect costs:** Loss of customers, the more firms rely on trade credit, the bigger the cost due to loss of suppliers will be, the loss of employees.

## Tradeoff theory

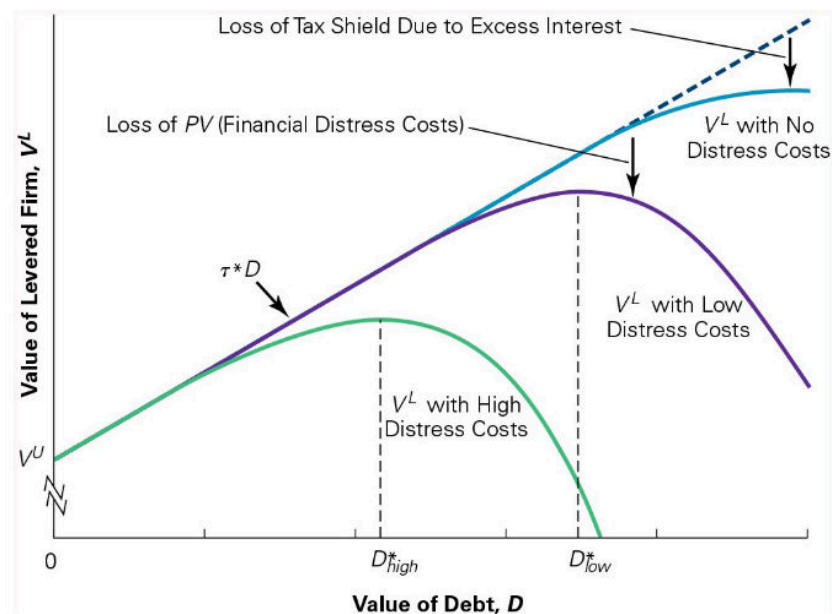
According to tradeoff theory, the firm will balance equity and debt in its capital structure by taking into account the benefits of the tax shield and the costs of financial distress.

Therefore, the total value of a levered firm is equal to the value of unlevered firm plus the present value of interest tax shield minus the present value of financial distress costs.

$$V^L = V^U + PV(\text{Interest Tax Shield}) - PV(\text{Financial distress costs})$$

The present value of financial distress costs is determined by:

1. The probability of financial distress
2. Magnitude of the costs
3. The discount rate of distress costs



## Agency costs

Agency costs are costs that are incurred when there are different interests between the holders of a firm. The costs will be created due to the conflicts between debt and equity holders that need to be solved due to the amount of leverage a firm chooses.

## Excessive risk-taking

When a firm is in financial distress it may choose to engage in very risky activities with a small probability of large gains. In a firm, shareholders will be the owners of a firm, but they bet on risk with the money of the debt holders, which is known as **risk shifting**. Due to this, shareholders will take more risky assets instead of low-risk ones,

for example by implementing a project with negative NPV, which is known as the **asset substitution problem**.

## Under-investment

When a firm is in financial distress it may also choose not to invest in a new low-risk and NPV-positive project. Because part of the project gains will go to debt holders to resolve the financial distress, NPV to equity holders will be reduced and may even be negative, thus the shareholders would decline the project. The agency cost of debt reflected is the **debt overhang**.

## Cashing out

Shareholders may liquidate assets below the market value when a firm with a lot of debt is on the verge of bankruptcy to pay out the proceeds as dividends.

## Leverage ratchet effect – Summary agency cost

**Leverage ratchet effect:** companies find it easier to increase leverage but much harder to reduce it.

When an unlevered firm issues new debt, equity holders bear any anticipated agency or bankruptcy cost through a discount in price they receive for that new debt. However, once a firm has debt already in place some of the agency or bankruptcy cost of taking additional leverage fall on existing debt holders.

1. Shareholders may have an incentive to increase leverage even when this does decrease the value of the firm.
2. Or the shareholders will not decrease leverage by buying back, even though this would increase the firm value.

If a firm would start issuing more debt, the  $r_D$  will increase, wealth will be transferred from debt holders to equity holders. Even though the firm value is reduced by increasing leverage, it does benefit shareholders.

## Debt maturity and debt Covenants

To mitigate agency costs of debt, debt holders can either issue shorter term debt or use restrictions such as **debt covenants** (limit the firm's ability to pay large

dividends, the types of investment the firm can make and the amount of new debt the firm can take on)

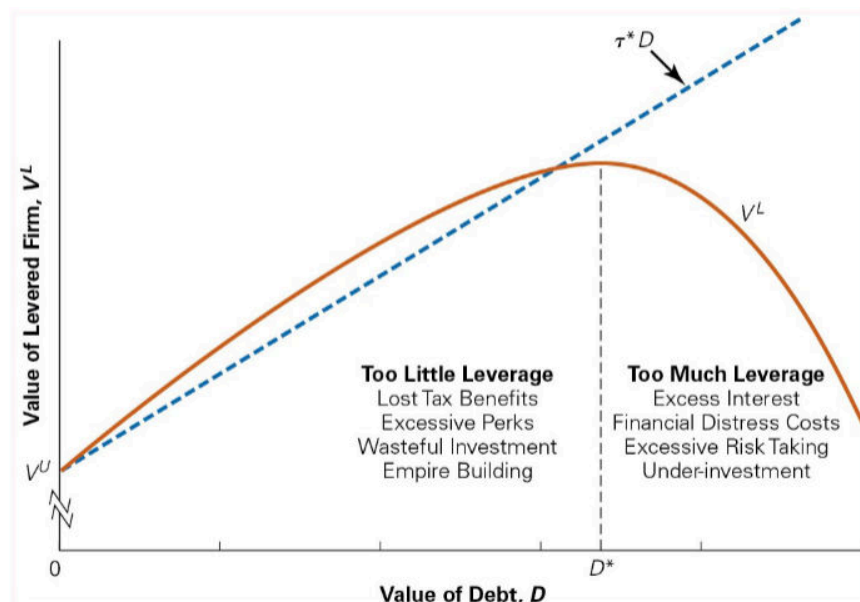
## Optimal leverage with Taxes, Financial distress and Agency costs

For the optimal level of debt there will be a trade-off between the tax shield of debt against the costs of financial distress and agency costs, which is known as the trade-off theory

$$V^L = V^U + PV(\text{Interest Tax Shield}) - PV(\text{Financial distress costs}) - PV(\text{Agency cost of debt}) + PV(\text{Agency benefits of debt})$$

### Agency benefits

- Maintain ownership concentration (equity stake)
- Reduce wasteful spending, which is more likely to when firms have excess cash (free cash flow hypothesis problem)
- Increase commitment (managers use more effective strategies when there's threat of financial distress)



## The optimal debt level

**R&D intensive firms:** Firms with high R&D expenditures and low current free cash flow. Because of a low EBIT the tax shield needed is very small thus there is only a little bit of debt.

**Mature firms:** usually have stable cash flows, tangible assets and fewer investment opportunities. Because tangible assets will have a high liquidation value, the financial distress costs will be low. The optimal level of debt for mature firms will be higher than that of growing firms

## Information

**Asymmetric information** is existent when the managers of a firm for example have superior information compared to investors about the future cash flows of a firm. With asymmetric information the parties will not all have the same information thus the assumptions that securities are fairly priced are not accurate any more. The investors will get this information via press releases, and penalties will be there for when intentionally no or wrong information is given.

The credibility principle states that an action will only be credible if the costs of taking this action would be too costly if the event was not true. When debt is used to signal some kind of information to investors, this is called the **signaling theory of debt**.

## Pecking-Order theory

Similarly to trade-off theory, **Pecking-Order theory** explains the choice capital structure of a firm. In this case, the key assumption is asymmetric information, meaning the management of the company is informed better about the prospects, risks and future profitability of the company than outside capital providers. Because of that, capital providers tend to ask for a higher premium to compensate for information asymmetry.

The signaling effect then causes the pecking order of financing, in which the firm will **use the cheapest form of financing first**. In this theory, using retained earnings is the cheapest way of financing with a lowest degree of asymmetric information, whereas issuing new shares (new equity) is considered the most expensive and used only if other sources are already exhausted.

*Internal financing → New debt → New equity*

## Issuing equity

A seller can give some information via the way he is selling his product. Because the bigger his desire to sell, the higher the probability the product is of low quality.

**Adverse selection** is when buyers and sellers have different information about the product. The **Lemons principle** states that when sellers have private information about the product and its value, buyers are likely to ask for lower prices because they do not know of which quality the product will be.

**The Lemons principle** implies that firms who start issuing new equity will sell these stocks at a discounted price to investors because investors are not willing to pay this much due to the possibility that bad news will be announced.

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