

# EFR summary

Finance 1, FEB12003X

2025 – 2026



Lectures and Exercise Lectures 1 to 5  
Weeks 1 to 5

**Deloitte.**

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## **Details**

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# Finance 1 – IBEB – Lecture 1, week 1

## Finance overview

Price is what you pay today while value is what you receive in the future, and the difference of the 2 determines your return

- The future value is uncertain, meaning there is risk, where the higher the risk a higher expected return is required

## Conceptual pillars in Finance

### 1. Financial markets connect savers and firms (investors)

- It moves money to most efficient projects
- Connect people with money to people who need money (nowadays through platform)
- Markets are powerful often very efficient, where price reflect knowledge of millions of investors (at that moment) and helps companies grow and create jobs

### 2. Capital is allocated based on risk and expected return (Investment decisions)

- How do companies decide which projects to invest in, finance tries to solve this using
  - o The trade-off between risk and return
  - o Time value of money: Trade-offs across time
  - o Financing choices: own funds, equity or debt

### 3. Markets allow risk sharing and diversification

- Spreading risk by owning shares of different companies instead of only one helps reduce risk, thus lowering cost of investing and more projects become viable
- At the household level, diversification helps save and invest for the long run

### 4. Financial institutions are the intermediate between savers and firms (investors)

- In the real-world funding from shareholders to firms is not direct, there are intermediaries: Banks, Private equity, Other financial institutions
- Regulation and oversight is a must to enforce rules

# Types of financial statements

**Balance Sheet.** Firm's financial position at a point in time (firm's assets and liabilities)

**Income Statement.** Firm's earnings (firm's revenues and expenses)

**Statement of Cash Flows.** Indicates the amount of cash generated by the firm.

**Statement of Stockholders' Equity.** Breaks down the stockholders' equity into issuing shares and retained earnings.

## Statement of cash flows

Usually, **Net income** is not equal to the amount of cash the firm has earned due to:

- Non-cash expenses (e.g. Depreciation & Amortization) which are taken into account in the net income, but you do not actual pay cash for it
- Uses of cash not on the income statement (e.g. Investment in PPE), meaning decrease in actual cash earned
  - o Capital Expenditure
  - o Change in (Current assets – Current liabilities)

**Free cash flow (FCF)** – the cash flow available for the company to repay creditors, pay dividends and interest to investors.

- The statement of cash flow includes **3 sections:** Operating activity, investing activity, and financing activity

Factor	Location
+ EBIT x (1- Tax Rate)	Income Statement
+ Non-cash Expenses (Depreciation, Amortization, etc.)	Income Statement
- Change in (Current Assets - Current Liabilities)	Balance Sheet (current period and previous period)
- Capital Expenditures (CAPEX)	Balance Sheet: Property, Plant, and Equipment (current period and previous period)
= Free Cash Flow	

# Valuation indicators

## Book value versus market value

**Book value:** how accountants evaluate a firm based on the sum of the net profits that were not paid out as dividends over the lifetime of the company. This is the book value of the firm's equity (Equity = Assets - Liability).

Financial economists, on the other hand, assess the value of a firm's equity by looking at its **market value** (or market capitalisation). This equals # shares outstanding \* stock price.

$$\text{market-to-book (MB) ratio} = \text{market value of equity} / \text{book value of equity}$$

When creditors and shareholders have more positive views on the firm's future than the suggestion from its book value, MB is often greater than 1. An obvious example is the potentially huge difference between the market value and the book value of a (great) football player.

**Enterprise value:** the market value of the firm's (underlying) assets that generate cash flows. This is the cost one needs to pay when taking over the enterprise.

$$\text{enterprise value (EV)} = \text{market value of equity} + \text{debt} - \text{cash}.$$

Cash here is the excess cash that is not needed for the firm's operating activities and can be paid back to investors without harming the business. This is different from working capital (cash needed to run the firm).

# Fundamentals in Finance

## Risk-return relations

Risk is the chance that the outcome is different than expected (care more about losses)

- The higher the risk, the higher the required return.

E.g. You can invest in 2 projects, investing \$ 950 today gives you

1. \$1000 in 2 years with 100% certainty
2. \$ 2000 in 2 years with 50% probability or \$ 0 with 50%

Both have the same expected outcome. But one guarantees (profit of 50) and the other is uncertain (profit of 1050 or loss of 950)

- Project 2 is riskier, and generally investors are risk-averse (don't like risk)

## No arbitrage

**Arbitrage:** without taking risks, you make a profit

**No arbitrage:** you cannot make a profit without taking risks

E.g. You can buy McDonalds chicken meal for a total of \$ 4.99, and then sell the different item individually, fries, coke and nuggets, this gives you a total of \$ 6.01.

- Meaning a profit of  $\$6.01 - \$4.99 = \$1.02$

This is arbitrage profits, however there are limits to Arbitrage

- May not exist for long:
- Transaction costs
- Hidden risks
- Opportunity costs

Due to the limitations above we have the No-arbitrage principle, where risk free profit opportunities cannot persist, this implies the following

- If 2 assets deliver the same cash flows in all states, they must have the same price, '**Law of one price**'
- Intuitively, if prices differed investors could buy the cheaper asset and sell the more expensive one, trades like this with time will eliminate the price difference
- This analogy forms the foundation for pricing, valuation, and relative comparison of assets

**Law of one price:** If equivalent investment opportunities trade simultaneously in different markets, then the price of trading should be the same everywhere. If this law does not hold, then an arbitrage opportunity exists.

# Finance 1 – IBEB

## Exercise lecture 1, week 1

### Introduction to financial statement analysis

#### Firms' disclosure of financial information

Financial statements are firm-issued accounting reports with past performance information. They are filled with the SEC (Securities and Exchange Commission). Financial statement analysis is used to compare the firm with itself over time, and compare the firm to other similar firms.

#### Balance sheet

A firm's balance sheet is a snapshot in time of the firm's financial position. The balance sheet identity is given by:

$$\text{Assets} = \text{Liabilities} + \text{Stockholders' Equity}.$$

**Assets:** what the firm owns.

- **Current assets:** cash or assets that are expected to be turned into cash within a year. This category includes cash, marketable securities (short-term low-risk investments like government bonds), accounts receivable, inventories, and other current assets such as pre-paid expenses.
- **Long-term assets** include net property, plant, and equipment (book value = cost of acquisition - accumulated depreciation), goodwill and intangible assets and other long-term assets, such as investments in long-term securities.

**Liabilities:** what the firm owes.

- **Current liabilities** are to be paid within a year. This includes accounts payable, short-term debt/notes payable, current maturities of long-term debt, and other current liabilities such as taxes payable, wages payable.

**Net working capital** is the capital that is available in the short term to run the business:

$$\text{Net working capital} = \text{current assets} - \text{current liabilities}.$$

- **Long-term liabilities** consist of other liabilities with the maturity of longer than one year and include long-term debt, capital leases, and deferred taxes.

**Stockholders' Equity:** the difference between the value of the firm's assets and liabilities.

- **Book value of equity** can be negative because it is calculated as the difference between book value of assets and book value of liabilities. However, many of the firm's valuable assets may not be reflected in the balance sheet (for example: the firm's reputation).
- **Market value of equity** (Market Capitalization) = Market price per share × number of shares outstanding. This cannot be negative and often differs substantially from book value.
- **Market-to-book ratio** (or Price-to-book ratio)
 
$$\text{market-to-book (MB) ratio} = \text{market value of equity} / \text{book value of equity}$$
 Value stocks:  $MB \text{ ratio} < 1$   
 Growth stocks:  $MB \text{ ratio} > 1$
- **Total enterprise value** (TEV)
 
$$\text{enterprise value (EV)} = \text{market value of equity} + \text{debt} - \text{cash}$$

## Income statement

Income statement indicates the flows of revenues and expenses over a period of time.

- An important component of an income statement is the "bottom line" (net income = earnings in a period).

### Earning calculations:

Total sales/revenue

-

Cost of sales

-----

Gross profit

-

Operating expenses



-----  
Operating income

+/-

Other income/expenses  
-----

Earnings before interest and tax (EBIT)

+/-

Interest income/interest expenses  
-----

Pre tax income

-

Taxes  
-----

Net income

Net income/N° of shares outstanding = EPS

## Statement of Cash Flows

Net income typically does not equal the amount of cash the firm has earned, because it includes non-cash expenses such as depreciation and amortization, and excludes cash uses such as investment in property, plant, and equipment or expenditure on inventory.

A statement of cash flows can be used to calculate free cash flows (FCF) and enterprise value. It includes three sections:

**1. Operating Activity:** Adjusts net income for all non-cash items related to operating activities and changes in net working capital.

Adjustments:

- Depreciation / amortization: add the amount of depreciation / amortization (as a non-cash expense)
- Account receivable: deduct the increases (as the cash is not yet been received)
- Accounts payable: add the increases (cash have not been paid yet)
- Inventories: deduct the increases (any increases in inventory are paid by cash)

**2. Investment Activity:** all cash required for investment activities

- Capital expenditures (purchasing PPE)

- Trading of marketable securities
- Acquisition related expenditures

### 3. Financing Activity:

- Payments of dividends (cash outflow, therefore is deducted)  
**retained earnings** = net income - dividends
- Changes in borrowings (increases in borrowings are cash inflows)

## Financial statements analysis & ratios

These financial statements are used to:

- Compare the firm to itself over time
- Compare the firm to other similar firms

We can derive different financial ratios from these statements to assess different factors of the firm

- **Profitability ratios:** e.g. Gross margin ratio
- **Liquidity ratios:** e.g. Current ratio
- **Working capital ratios:** e.g. Account receivable/payable days
- **Interest coverage and leverage ratios:** e.g. EBIT interest coverage ratio, debt-to-enterprise value ratio
- **Valuation ratios:** e.g. Enterprise value to EBITDA

## Financial decision-making and law of one price

Decision rules based on benefits and costs need to be made at the same moment in time

- When we express the value in terms of dollars today, we call it the **present value (PV)** of the investment
- When we express the value in terms of dollars in the future, we call it the **future value (FV)** of the investment

**Financial decision making:** Investment should be made when  $PV(benefits) > PV(costs)$ .

$$NPV = PV(benefits) - PV(costs) \rightarrow NPV > 0$$

**Arbitrage** refers to taking advantage of the price difference when buying and selling equivalent goods in different markets. An arbitrage opportunity occurs

when it is possible to make a profit without taking any risk or making any investment.

**Normal market** is a competitive market in which there is no arbitrage opportunity.

**Law of One Price:** If equivalent investment opportunities are traded at the same time in different normal markets, then they must trade for the same price in both markets.

# Finance 1 – IBEB Lecture 2, week 2

## Time value of money

Financial decisions are often made by comparing values:

1. Values can be compared only at the same point in time
2. Compound cash flow to move it forward in time

$$FV_n = C * (1 + r)^n$$

3. Discount cash flow to move it backward in time:

$$PV = \frac{C}{(1 + r)^n}$$

## Valuing a stream of cash flows

### Present value of a cash flow stream

$$PV = \sum_{n=0}^N \frac{C_n}{(1 + r)^n}$$

### Future value of a cash flow stream

$$FV = PV(1 + r)^n$$

### Annuities – Fixed period.

- Constant cash Flow or cash flow with constant growth

$$PV(\text{annuity with growth}) = \frac{C}{(r + g)} \left[ 1 - \left( \frac{1 + g}{1 + r} \right)^N \right]$$

### Perpetuities – Infinite life.

- Constant cash Flow or cash flow with constant growth

$$PV(\text{perpetuity}) = C/r$$
$$PV(\text{growing perpetuity}) = C/(r - g)$$

**Note:** Infinite life, but not infinite value if  $g < r$ , usually this is the case

- Both Annuities and Perpetuities start 1 year from today, so if the question specify that it starts, say 2 years from today then you must discount it once more

$$\frac{1}{1 + r} \cdot \frac{C}{r}$$

## Discounting with the risk-free rate

**Risk-free interest rate,  $r_f$ :** interest rate at which money can be borrowed or lent without risk over that period

If future payments are risky, premium needs to be added to interest rate to account for riskiness (higher risk => higher interest rate).

## Financial decision-making with NPV

Cost-benefit analysis for an investment opportunity can be done by calculating the Net Present Value (NPV) using the formula:

$$NPV = PV(benefits) - PV(costs)$$

**NPV decision rule:** invest in the alternative with the highest NPV. Choosing this alternative is equivalent to receiving its NPV in cash today.

The net present value of a stream of cash flows can be valued by summing the discounted values of each future cash flow with the appropriate interest rates regarding the time distance.

**E.g.** you give me EUR 100 now. After one year, I'll give you EUR 107.  $r_f$  is 5%

- $NPV = 107 / (1+5\%) - 100 = 101.90 - 100 = 1.90$
- This is a good deal as NPV is positive. This reflects that the 107 is more than the 105 that you'd invest the 100 on a bank account

## Valuing bonds

Bonds cashflows amount and timing tend to be known in advance (they are **fixed**)

- It's a security issued by a borrower (e.g. government or company) and purchased by an investor.
- The bond issuer is raising money (borrowing) today
- The investor has delivered some money to the issuer (a loan) and expects to see regular cashflows repaying this money over time

Cash flows:

- Investor pays the bond Price
- Issuer pays the pre-specified payments: Coupons and final repayment

**Note:** We need to use present value techniques to compare cash flows

## Two types of bonds

**1. Zero coupon bond:** Offers a single payment.

- Purchaser pays the price of the bond
- The bond promises in K-year to pay the bondholder a single payment, called **face value** (par value)
- The date of this payment is called the **maturity date**. K is the time to maturity

**2. Coupon bond:** Zero coupon bonds with additional periodic payments

- The bond promises in K-year to pay the bondholder a single payment, called face value
- The date of this payment is called the maturity date. K is the time to maturity
- Additional coupons: At regular intervals until maturity, the bondholder receives a coupon payment
- Coupon rate: the ratio of total annual coupon to face value

## Bond price: present value of cash flows

**Step 1:** Identify all cash flows

**Step 2:** Discount these cash flows using rules of time travel. For very long-term bonds, we can use the annuity formula for the coupons + present value of final payment of face value

- Annuity without growth

$$PV_A = \frac{C}{r} \left[ 1 - \left( \frac{1}{1+r} \right)^T \right]$$

- Bond value = PV (coupon value) + PV (face value)

$$PV_B = \frac{Coupon}{r} \left[ 1 - \left( \frac{1}{1+r} \right)^T \right] + \frac{Face\ value}{(1+r)^T}$$

## Examples

The following tables show the Cash flow of the following bonds

- Five-year coupon bond with face value of 1000, coupon rate and interest rate 5%
- Five-year zero-coupon bond with face value of 1000, interest rate is 5%
- Reversed question: If the price is 1000 of a zero-coupon bond with a 5-year maturity
  - What is the face value?

	CF Coupon	CF Face value	PV		CF	PV		CF	PV
today			1000.00			783.53			1000.00
t = 1	50		47.62			0.00			0.00
t = 2	50		45.35			0.00			0.00
t = 3	50		43.19			0.00			0.00
t = 4	50		41.14			0.00			0.00
t = 5	50	1000	822.70		1000	783.53		1276.28	1000.00

## What drives bond prices

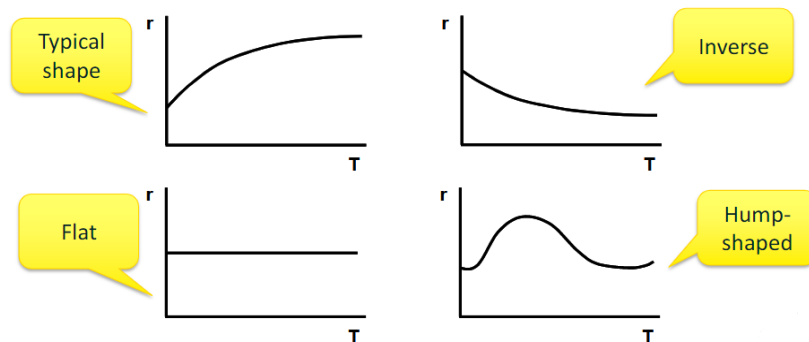
### 1. Interest rate

If after the issue of a bond, the general level of discount rates in the economy rose then we would expect bond prices to fall.

- When a bond is issued the coupon rate should reflect the current market conditions (interest rates)
- **Note:** cashflow to be received at different future dates should be discounted using different discount rates
  - We call these the #-year spot rates

**Term structure** of interest rates indicates how interest rates vary with the maturity at one point in time.

- The most typical case depicts an upward-sloping term structure, because the longer the time until maturity the higher the interest rate.
- But there can also be different variations



## 2. Risk of default

The possibility of default adds a credit spread, increasing the required return and lowering the bond's price.

Credit spread is a risk premium, that compensates for taking risk

$$Price = \sum_{t=1}^T \frac{Face\ value}{(1 + r_f + credit\ spread)^t}$$

## Bond price: yield to maturity

We often use Yield to Maturity rather than the bond price because,

- Bond prices may differ because of coupon rates
- YTM is an annual rate and much more comparable

**Yield to Maturity** is the constant, hypothetical discount rate that equates the PV of the bonds cashflow with it's price

- Basically, we solve for the YTM and get a fixed rate

High bond price must mean a low YTM and low price means high YTM

$$Price = \frac{C_1}{(1 + YTM)} + \frac{C_2}{(1 + YTM)^2} + \dots + \frac{C_n}{(1 + YTM)^n} + \frac{Face\ value_n}{(1 + YTM)^n}$$

If we know the price, we can calculate the yield (and vice versa)

YTM is the weighted average of all spot rate used for discounting, this because different spot rates are discounted more or less, depending on the year

$$Price = \sum_{t=1}^T \frac{C_t}{(1 + r_t)^t} = \sum_{t=1}^T \frac{C_t}{(1 + YTM)^t}$$



# Finance 1 – Exercise lecture 2, week 2

## Time value of money

It is not possible to simply sum or subtract cash flows that occur at different points in time, this is because of time value of money, as we have seen last week, we have to express all cash flows at the **same point in time**

- Moving cash flow backward, by expressing all future cash flows' values by **discounting** them to today
- Moving cash flow forward, by expressing all cash flows at a period later than today by **compounding** them

So, to calculate the PV of a stream of cash flows you should discount them to today, but if you have many cash flows this can take quite some time

- For this we have 4 short-cut formulas

## PV shortcut formulas

$g$  = growth rate,  $n$  = number of periods,  $c$  = cash flow,  $r$  = interest rate

Type of cash flows	Constant cash flows	Growing cash flows
Perpetuities (last forever)	$g = 0, n \rightarrow \infty$ $PV (perpetuity) = \frac{C}{r}$	$g < r, n \rightarrow \infty$ $PV (growing perpetuity) = \frac{C}{(r-g)}$
Annuities ( $N$ periods)	$g = 0, n \rightarrow N$ $PV (annuity) = \frac{C}{r} \times \left[ 1 - \frac{1}{(1+r)^N} \right]$	$n \rightarrow N$ $PV (growing annuity) = \frac{C}{r-g} \times \left[ 1 - \left( \frac{(1+g)}{(1+r)} \right)^N \right]$

**Note:** If you know the formula for growing annuities you know them all

## Interest rates

**Effective Annual Rate (EAR)** indicates the total amount of interest that will be earned at the end of one year.

- Typically used in present value calculations for yearly cash flows as it considers the effect of compounding.
- Also referred to as the Effective Annual Yield (EAY) or Annual Percentage Yield (APY).

**Annual Percentage Rate (APR)** indicates the amount of simple interest earned in one year.

- **Simple interest** is the amount of interest earned without the effect of compounding.
- The APR is typically less than the EAR.

**Note** that the APR cannot be used as a discount rate without adjustments made.

The APR with  $k$  compounding periods is a way of quoting the actual interest earned each compounding period:

$$\text{Interest Rate per Compounding Period} = APR / \left( \frac{k \text{ periods}}{\text{year}} \right)$$

To convert an APR to an EAR, we can use the following formula:

$$1 + EAR = \left( 1 + \frac{APR}{k} \right)^k$$

- The EAR increases with the frequency of compounding, where **continuous compounding** is compounding every instant

## Valuing bonds

**Bond:** A tradable loan, which generally has a fixed maturity and fixed coupon payments (cash flow stream).

### Two types of bonds

We have Coupon bond and Zero-coupon bonds, the main difference is that one only pays out a **face value**, but the other has additional **coupon payments**, this is explained more precisely on Lecture 2 of week 2

**Yield to maturity:** The discount rate that sets the present value of the promised bond payment equal to the current market price of the bond

## Relation between spot rate and forward rates

An **interest rate forward contract** or **forward rate agreement** is a contract today that fixes the interest rate for a loan or investment in the future.

Suppose we have the following options:

- Invest in a risk-free 2 year 0-coupon bond
- Invest in a risk-free 1 year 0-coupon bond and a 1 year forward contract for the second period

By the law of one price these 2 must have the same return

$$(1 + r_2)^2 = (1 + r_1)(1 + f_2)$$

- Where  $f_2$  = forward rate,  $r_1$  = interest rate for the 1 year,  $r_2$  = interest rate for the 2 year

# Finance 1 – IBEB – Lecture 3 & Exercise lecture 3, week 3

## Investment decision rules

### 1. Net present value (NPV)

$$NPV = \sum_{t=0}^T \frac{CF_t}{(1 + R_t)^t} = -INV_0 + \sum_{t=0}^T \frac{CF_t}{(1 + R_t)^t}$$

- Where INV, is the initial investment

The NPV measures economic value creation, so  $NPV > 0$  is attractive, we choose the investment with the highest NPV

#### Complexities in Using NPV

1. Comparing projects with varying durations.
2. Capital constraints

#### Duration of the projects

For ongoing projects, a fair comparison of NPV can only be done when periods are equal, for example:

- Project A: Invest 50 today to earn 80 per year in the next three years
- Project B: Invest 50 today to earn 100 per year in the next two years

**First**, we need to extend the projects until the duration is equal, **then** we calculate the NPV of both in the extended form

Project A:

	T=0	1	2	3	4	5	6
A CF	-50	80	80	80			
A2 CF				-50	80	80	80

Project B:

	T=0	1	2	3	4	5	6
B CF	-50	100	100				
B2 CF			-50	100	100		
B3 CF					-50	100	100

As you can see we have extended the duration to 6 years by repeating the investment, doing so gives the following NPV

- A + A2: NPV of 261mIn €

- B + B2 + B3: NPV of 310mln €

So, we choose Project B, unlike without adjusting for time we would have chosen A

## Capital constraints

Sometimes firms have many positive NPV projects and they can't invest in all of them simultaneously

To tackle this, we take the following steps:

Step 1: Form all possible combinations between available projects

Step 2: Cancel those combinations that are impossible due to limited budget

Step 3: Choose the combination with the highest NPV

We have projects A, B & C each costing 100, but we only have 200€, and **no negative cash flow is allowed for t = 1 and t = 2**

	A	B	C	A&B	A&C	B&C	A&B&C	Nothing
t = 0	-100.0	-100.0	-100.0	-200.0	-200.0	-200.0	-300.0	0.0
t = 1	109.0	0.0	-105.0	109.0	4.0	-105.0	4.0	0.0
t = 2	2.0	123.0	400.0	125.0	402.0	523.0	525.0	0.0
NPV (@10%)	0.7	1.7	135.1	2.4	135.9	136.8	137.5	0.0

- In this case we see that it is best to do A&C combined

The example above is a simple one with only 3 project which can be done manually, but when we have many projects and constraints, lots of computing power is needed to solve the problem

- We have simple heuristics that could work, profitability index

**Profitability index** (NPV per constrained resource) = NPV / Investments

It can be used to identify the optimal combination of projects to invest in under the budget constraint. Rank projects by PI and select the ones with the highest indicator.

$$PI = \frac{\text{value created}}{\text{resource consumed}} = \frac{NPV}{\text{resource consumed}}$$

**Note:** It needs to meet 2 important constraints

- Set of projects taken following the PI ranking should completely exhaust the available resources
- There is only a single relevant resource constraint

### Strengths of using NPV

- Often right and unambiguous
- Incorporates time value of money
- Easy to compare between projects
- Takes investment size into account
- Can compare one large project versus several smaller ones

### Weakness:

- Needs an appropriate discount rate

The **correct methods** for the right investment decision are the ones we talked about above, alternative methods may be more practical in some circumstances, but may be wrong

## 2. Payback period

**The payback period (PBP):** The amount of time it takes to recover or pay back the initial investment

A firm must invest £150M to set up a factory that will sell products forever, earning £10M annually. The interest rate is 7%

- Payback period =  $150/10 = 15$  years

Criteria to invest depends on what is set by the firm

- **Invest**, if payback period < required payback (set by the firm)
- **Do not invest** if payback period > required payback, e.g. when required payback = 10 years
- Choose the one with shortest payback period when there is options
- The formula below tell us that the payback period is the first moment when total cash inflows become **greater than or equal to** the initial investment

$$INV_{t=0} \leq \sum_{t=1}^{PBP} CF_t$$

### Pitfalls of the PBP method

- It ignores cash flows after the payback period
- It ignores time value of money

### 3. Internal rate of return (IRR)

**Internal rate of return (IRR)** is the interest rate at which the NPV of an investment project equals zero.

A firm must invest £150M to set up a factory that will sell products forever, earning £10M annually. The interest rate is 7%

- The NPV is equal to 0 when,  $(10 / \text{IRR}) - 150 = 0$
- $\text{IRR} = 6.6\%$

Criteria for investing

- Invest, if  $\text{IRR} > \text{cost of capital}$  (if the IRR is larger than the true discount rate)
- Do not invest, if  $\text{IRR} < \text{cost of capital}$

When choosing among several projects, choose the one with the **highest IRR**. But firms have good reasons not to base their decision on the highest IRR, due to the following reasons

- It ignores the size of a project, and with longer duration projects where reinvestment is assumed this might be a disadvantage.
- In case of positive and negative cash flows there is a possibility for multiple IRRs to exist with positive and negative interest rates when NPV is zero.

In the example below you can see the calculations with all 3 methods

- For Project A we see that using the PBP it shows that it only requires 1 year, but actually the NPV shows that it's negative, this is because **PBP ignores cash flows after the payback period, in this case it was negative**
- For Projects B & C, we see that their IRR are very similar but their NPV are vastly different, this is because **IRR ignores scale**, meaning it's only a ratio without capturing the size

Project	t = 0	t = 1	t = 2	t = 3	t = 4
A	-10	10.5	0	0	0
B	-100	0	0	0	300
C	-10	0	0	0	31

PBP	IRR	NPV (@10%)
1	5.0%	-0.45
4	31.6%	104.90
4	32.7%	11.17

# Capital budgeting

When we decide to invest in a project, we want to know how our total cash flows change because of that project

- For that we need incremental earnings, which is different from total earnings

Capital budgeting is about:

- Forecasting free cash flows
- Understanding incremental earnings
- Dealing with uncertainty

Using the formulas below it is easy to calculate the FCF for historical data, the difficult part and what analysts spend most of their time on is forecasting

$$\text{net working capital} = \text{current assets} - \text{current liabilities}$$

$$\text{Free Cash Flow} = (\text{Revenues} - \text{Costs} - \text{Depreciation}) \times (1 - \tau) + \text{Depreciation} - \Delta\text{NWC} - \text{CapEx}$$

**Incremental** cash flows = the difference between doing and not doing the project.

**Cannibalization** occurs when the introduction of a new product has an adverse impact on the sales of existing products.

**Opportunity cost** – the revenue that could have been earned with an alternative use of the asset

- Think of machines that you own but are currently **idle**, you start a new project and use them, but this is not free, there is an opportunity cost, like selling the machines, these are earnings that we won't receive if we use it for the project
- Remember we care about **incremental earnings**, not doing the project gives you cash flow from selling the machine, so this is a cost

**Sunk costs** – unrecoverable costs that are already incurred and therefore irrelevant for the decision making.



## Dealing with uncertainty

- **Break-even** analysis: this analysis finds the value of a parameter for which the  $NPV = 0$ . After this we can determine how likely it is that the parameter value is below or above the break-even amount
- **Sensitivity** analysis: input assumptions in this analysis will be changed within a plausible range, which will affect the NPV.
- **Scenario** analysis: in this analysis, the values of parameters will be determined for multiple scenarios, like a best case and worst case. These scenarios will be compared by their NPV when multiple parameters are changed at the same time.

## Valuing stocks

For bonds, the cash flows were known, they consisted of coupons and a principal. But stock only pays dividends, we get the value of what is left of the earnings of a company, which are not known in advance.

### Terminology

- Common Stock: security representing a share in the ownership of a corporation.
- Initial Public Offering: the first sale of stock in a corporation to the public.
- Secondary Market: a market, often a stock exchange, in which previously issued shares are traded amongst investors.
- Dividends: payments made by companies to shareholders.
- Dividend yield: ratio of annual dividend to share price. There are different types (for example, stock or cash; preferred and common)
- P/E Ratio: share price divided by earnings per share (price-to-earnings)

Two ways to estimate the value of a stock: **Dividend Discount Model (DDM)** or estimating the **value using comparable firms**.

## Dividend Discount Model (DDM)

Price of today based on expected payments, dividends are also accounted for in the return

$$P_0 = \frac{div_1}{1 + r_E} + \frac{P_1}{1 + r_E} \Rightarrow r_E = \frac{div_1}{P_0} + \frac{P_1 - P_0}{P_0}$$

- where  $\frac{div_1}{P_0}$  represents dividend yield,
- and  $\frac{P_1 - P_0}{P_0}$  the capital gains rate.

Now since we know the formula for  $P_0$ , we can replace every "price" with a future price

$$P_1 = \frac{div_2}{1 + r_E} + \frac{P_2}{1 + r_E}$$

$$P_0 = \frac{div_1}{1 + r_E} + \frac{div_2}{(1 + r_E)^2} + \frac{P_2}{(1 + r_E)^2}$$

Now if we keep on replacing with future prices we will be left with only the dividend part, this is because, same as for perpetuity as long as the growth is lower than  $r_E$

$$P_0 = \sum_{i=1}^{\infty} \frac{div_{t+i}}{(1 + R_E)^i}$$

$$R_E = R_t + \text{equity risk premium}$$

### The **Gordon (growth) model**

Zero dividend growth (i.e. constant dividend)

$$P_t = \frac{D}{r_e}$$

Constant dividend growth with factor g

$$P_t = \frac{D}{r_e - g}$$

## Multiple valuation of comparable stocks

A company does not have to pay all earnings, they can keep them as retained earnings, which they can use for new investments.

$$div_t = \text{EarningsPerShare}_t \times \text{Dividend payout rate } (k)_t$$

Following the **Gordon growth model**

$$\text{Price}_{t=0} = \frac{\text{Dividend}_{t=1}}{Re - g} = \frac{k \cdot EPS}{Re - g}$$

By combining these formulas, we can arrive at P/E ratio:

$$\frac{Price_{t=0}}{Earnings} = \frac{k \cdot EPS}{Re - g} \cdot \frac{1}{EPS} = \frac{k}{Re - g}$$

Other multiples include:

- Earnings based: EV / EBIT(DA)
- Sales based: P / Sales or EV / Sales
- Asset-based: P / book value, EV / assets
- Cash flow based: P / cash flows
- Sector specific: Price / users, Price / subscribers, EV / capacity

## Alternative models for price stocks

### Total payout model

- $P_0 = \text{PV}(\text{Future total dividends and share repurchases}) / \text{shares outstanding}_0$

### Discounted FCF model

$$P_0 = \frac{(V_0 + \text{cash}_0 - \text{debt}_0)}{\text{shares outstanding}_0}$$

- Where,  $V_0 = \text{PV}(\text{FCF's})$

### Theories

- Efficient market hypothesis: Based on available information, it is not possible to systematically achieve abnormal returns
- Informational efficiency: All relevant information is fully reflected in market prices
  - Price adjustments as a result of new information are direct and correct, **No over/under reaction**
  - Price = value, **No over or undervaluation**

# Finance 1 – IBEB – Lecture 4 and Exercise lecture 4, week 4

## Valuing stocks

### Information

The first version of the three levels of informational efficiency was constructed by Fama in 1970:

- **Weak:** The stock prices reflect all information on its historical price. In technical analysis, trends of history are used to predict the future, but these cannot be considered useful.
- **Semi-strong:** the prices of the stock are based on the publicly available information which is relevant for pricing. The prices adjust to public information.
- **Strong:** not only is the publicly available information reflected in the prices, but also insider information of firms which is not yet public to all investors.

### Lessons from efficient markets

**Efficient market hypothesis** - when relevant information is available it is immediately and completely reflected in prices, therefore it is not possible to systematically achieve abnormal profits.

- For investors to consistently beat the market, competitive advantage is needed, with low transaction costs, no regulation, no benchmark index, etc.
- Arbitrage opportunities are rare and short-lived
  - There are companies that use LLM to trade within milliseconds when new information is out e.g. High frequency trading

Two competing views

- Efficient markets (Fama): prices reflect information quickly
  - Abnormal returns hard to predict
- Behavioral Finance (Thaler): psychology and limits to arbitrage can create systematic mispricing

To determine whether a market is efficient, we need a theory for risk and return.

# Measuring risk

## Risk and return measures

### Example

Stock has 3 possible returns: -10%, 5%, and 15% which happen with 1/3 probability

What is the expected return ( $\mu$ ), variance ( $\sigma^2$ ) and standard deviation ( $\sigma$ )?

- Expected return: weighted average of returns

$$\left(-0.10 \cdot \frac{1}{3}\right) + \left(0.05 \cdot \frac{1}{3}\right) + \left(0.15 \cdot \frac{1}{3}\right) = 3.3\%$$

- Variance: Weighted average of squared deviations from the expected return

$$\sigma^2 = \frac{1}{3} \cdot (-0.1 - 0.033)^2 + \frac{1}{3} \cdot (0.05 - 0.033)^2 + \frac{1}{3} \cdot (0.15 - 0.033)^2 = 0.014$$

The standard deviation or volatility is  $\sigma = 11.94\%$

**Expected return**, for a portfolio with N possible return and probabilities

$$E[R_p] = \sum_{i=1}^N P_i \cdot R_i$$

**Variance** - indicates how much the squared deviation from the mean is, which can be calculated with the formula:

$$\sigma^2 = \sum_{i=1}^N P_i (R_i - \bar{R})^2$$

The standard deviation,  $\sigma(R)$ , which is the square root of the variance, in Finance is also known as **volatility**.

## Historical returns

The example and formulas used in the previous page is based on outcome probabilities which are 'expected', thus we consider them the population

Instead of using expected outcomes, historical data which are 'realized' can be used to estimate risk and return

- However, more historical data, means more different world

- The main difference is that with historical data we are working with a sample

**Expected return**, this formula stays the same as before

**Variance**, we are working with a sample and not a population

- Suppose we have K years of annual returns on a stock, meaning K N° of observations

$$s^2 = \frac{1}{K-1} \sum_{i=1}^K (R_i - \bar{R})^2$$

### Example

What if I have 5 years of annual returns on say the S&P 500?

- Year 1: -35.6%, Year 2: 32.4%, Year 3: 16.5%, Year 4: 3.8%, Year 5: 13.8%.
- Average return: 6.2%

$$s^2 = \frac{1}{4} \cdot ((-0.356 - 0.062)^2 + (0.324 - 0.062)^2 + (0.165 - 0.062)^2 + (0.038 - 0.062)^2 + (0.138 - 0.062)^2)$$

$$s = \sqrt{0.065} = 25.5\%$$

## Portfolio risk and return

### What is a portfolio?

A portfolio is a collection of securities

- Every stock has it's own average risk and return
- To calculate the risk and return of the entire portfolio we need to factor in how these returns covary with each other

Main notations

- N is the number of assets in your portfolio
- $x_i$  is the fraction of your wealth allocated to each asset i (1 to N)
- $\sigma_i^2$  is the variance of the return on asset i
- $p_{i,j}$  is the correlation between the returns on stocks i and j

### Simple example

Say you have 10,000 euros allocated over 50 different companies

- You own 2 shares of ASML stock worth 1,150 euros per share

Portfolio weight is  $\frac{2 \cdot 1150}{10000} = 23\%$

## Portfolio average returns

Portfolio weights must sum to one:  $\sum_i x_i = 1$

But they can be positive, zero or even negative!

- In the stock-market: short sales
- Selling an asset you do not own!
- Repurchase at a later date and deliver it to the lender
- Can be done typically by professional investors or market makers

But many 'retail' investors have credit or lend their assets

$$E(R_p) = \mu_1 \cdot x_1 + \mu_2 \cdot x_2 + \dots + \mu_N \cdot x_N$$

$$E(R_p) = \sum_{i=1}^N \mu_i x_i$$

## Portfolio return variance

To calculate the variance of a portfolio  $Var(R_p)$

- The variance of each asset i in the portfolio:  $\sigma_i^2$
- The portfolio weight of each asset i:  $x_i$
- The covariance of the return of each asset with other asset:  $Cov(r_i, r_j) = \sigma_{i,j}$

You need to capture all these ingredients in one generalized formula, as we derive below for a two-asset case:

$$Var(R_p) = \sum_{i=1}^N \sum_{j=1}^N x_i x_j \sigma_{i,j}$$

This general formula for a two-asset case can also be written as:

$$Var(R_p) = x_1 x_1 \sigma_{1,1} + x_1 x_2 \sigma_{1,2} + x_2 x_1 \sigma_{2,1} + x_2 x_2 \sigma_{2,2}$$

- We know  $\sigma_{1,1} = Cov(r_1, r_1) = Var(r_1) = \sigma_1^2$  and  $\sigma_{2,2} = Cov(r_2, r_2) = Var(r_2) = \sigma_2^2$   
$$Var(R_p) = x_1^2 \sigma_1^2 + x_1 x_2 \sigma_{1,2} + x_2 x_1 \sigma_{2,1} + x_2^2 \sigma_2^2$$
- We also know that  $Cov(r_2, r_1) = Cov(r_1, r_2) = \sigma_{2,1} = \sigma_{1,2}$
- And  $\sigma_{1,2} = \rho_{1,2} \sigma_1 \sigma_2$  **because**  $\rho_{1,2} = \frac{Cov(r_1, r_2)}{\sigma_1 \sigma_2} = \frac{\sigma_{1,2}}{\sigma_1 \sigma_2}$

Final formula:

$$Var(R_p) = x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + 2x_1 x_2 \rho_{1,2} \sigma_1 \sigma_2$$

## Example

Consider two stocks with a correlation of 0.15

- Stock 1: Expected return 20%, standard deviation 30%, invested 100 million
- Stock 2: Expected return 12%, standard deviation 18%, invested 300 million

$$E(R_p) = 0.25 * 0.20 + 0.75 * 0.12 = 0.14 = 14\%$$

$$Var(R_p) = 0.25^2 * 0.30^2 + 0.75^2 * 0.18^2 + 2 * 0.25 * 0.75 * 0.15 * 0.30 * 0.18 = 0.0268$$

The portfolio standard deviation is 16.4%

- Lower than both individual assets, meaning lower risk

## Diversification

### Diversification in stock portfolios

The key theory that we follow to understand risk and return in portfolios is

**'Markowitz (1952) portfolio theory'**, where we assume

- Investors like return
  - If 2 investments have the same risk, but one has a higher mean return then, investor will choose the one with higher mean return
- Investors dislike risk
  - If 2 investments have the same Mean return, but one has a higher risk (std dev) then, investor will choose the one with lower risk

Diversification eliminates **'idiosyncratic risk'**

- Idiosyncratic risk = stock/firm-specific risk
- The smaller correlation between stocks the more opportunities for risk reduction

### Example

Suppose we have 2 stocks where correlation between their returns is 0.2

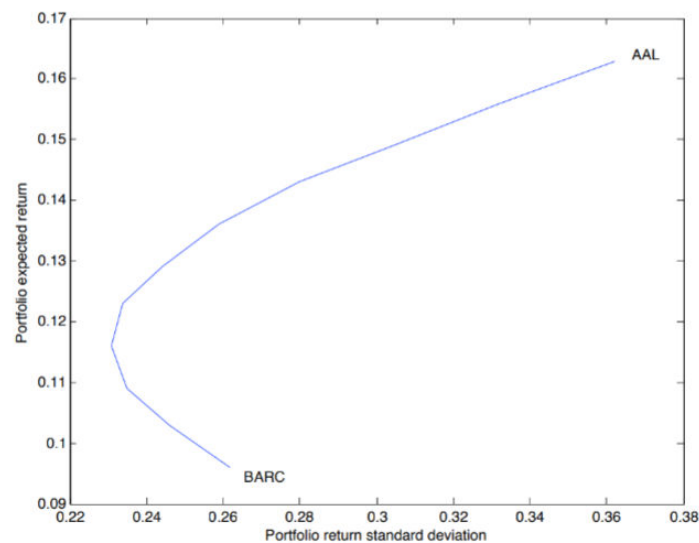
Stock	$E(R_i)$	$\sigma_i$
AAL	16.3%	36.2%
BARC	9.6%	26.2%

The following table shows what happens to the portfolio risk and return if we vary the weights

$x_{AAL}$	$x_{BARC}$	$E(R_i)$	$\sigma_i$
0	1	0.096	0.262
0.1	0.9	0.103	0.246
0.2	0.8	0.109	0.235
0.3	0.7	0.116	0.231
0.4	0.6	0.123	0.234
0.5	0.5	0.129	0.244
0.6	0.4	0.136	0.259
0.7	0.3	0.143	0.28
0.8	0.2	0.149	0.304
0.9	0.1	0.156	0.332
1	0	0.163	0.362



If we plot the Expected return and the return standard deviation, we get the '**Mean variance frontier**'

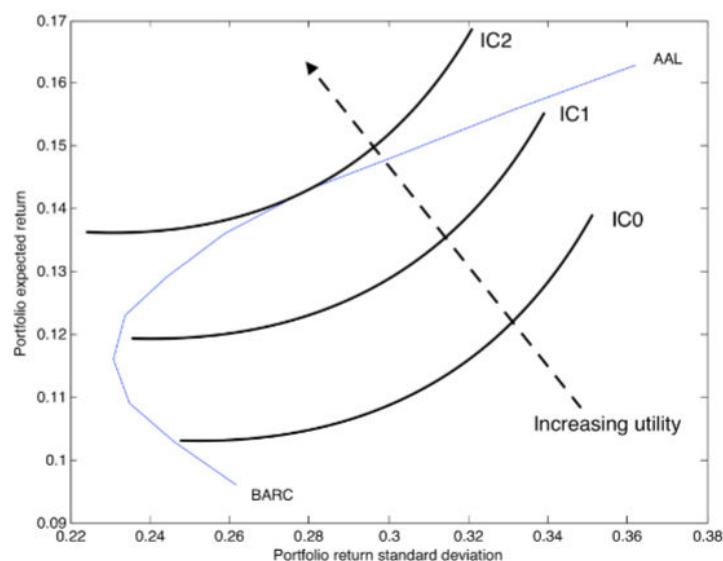


**Note:** one cannot infinitely reduce risk, this is because stock prices are not perfectly correlated with each other, meaning market-wide or 'systematic' risk shocks move all stocks to some degree

## Optimal portfolio choice

From the mean variance frontier plot we can see all the possible combinations of portfolio risk and return, based on the weights, typically investors like return but dislike risk, so we assume '**risk aversion**'

- Thus, we can plot the indifference curves based on his risk aversion and get the optimal portfolio (point of tangency)
- **Note:** indifference curve varies across preferences



So far, we have assumed that all assets are risky, if we add a risk-free asset

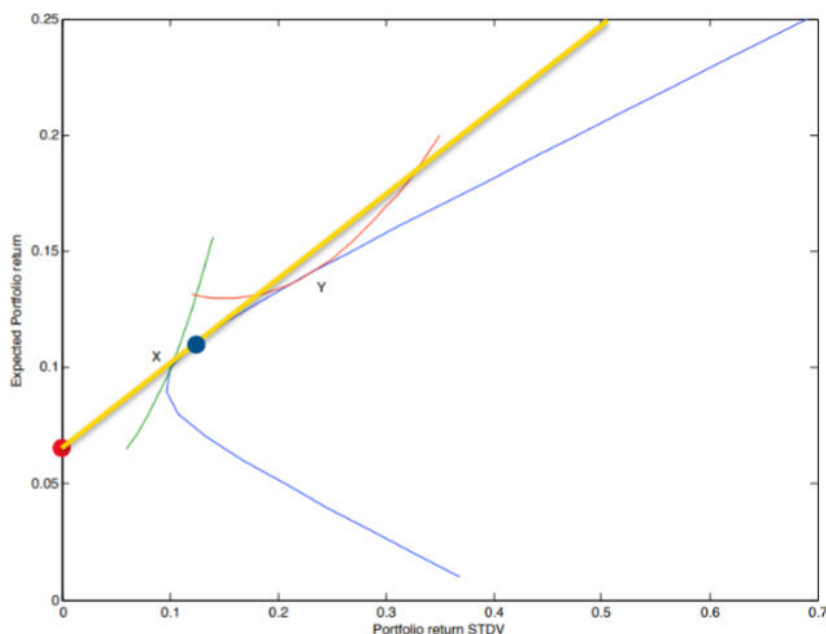
- Return known in advance
- No risk, so standard deviation is 0
- Return is uncorrelated to other assets that are not risk-free

In the graph below the

- Blue dot is the **tangency-portfolio** which tells us the optimal portfolio of risky assets (**Market portfolio**) that maximises return per unit of risk, based on the **Sharpe ratio**

$$\text{Sharpe ratio} = \frac{\text{Expected return} - \text{Risk free return}}{\text{Portfolio volatility}}$$

- Red dot represents the **risk-free asset**
- Yellow line is the **Capital market line**, which shows the highest possible combination of risk and return when you are allowed to combine risk-free assets and the market portfolio



## Capital Asset Pricing Model (CAPM)

CAPM tells us how stocks should be priced if we believe in Markowitz theory, it starts with the assumption that everyone holds the market portfolio and the risk-free asset, with the only difference being the weights

- The **Firm specific risk** can be diversified away
- However, **Market risk** requires compensation, because it's 'systematic' risk that cannot be diversified away

- This is measured by  $\beta$ , the sensitivity of the investment I to the fluctuations of the Market portfolio

So, if the only risk that matter is the Market risk and this is measured by  $\beta$ , then  $\beta$  will price all the stocks, according to CAPM

### The CAPM equation

$$E(R_i) = R_f + \beta_i(E(R_M) - R_f), \quad \beta_i = \frac{Cov(R_i, R_M)}{Var(R_M)}$$

- The risk-free rate  $R_f$ .
- The market risk premium:  $(R_m - R_f)$ 
  - $R_m$  is the market portfolio return
- $\beta_i$  measures asset  $i$ 's risk.
  - Taking more risk, means requiring more compensation
  - Expected return is linearly increasing in risk.

### Implications of $\beta_i$

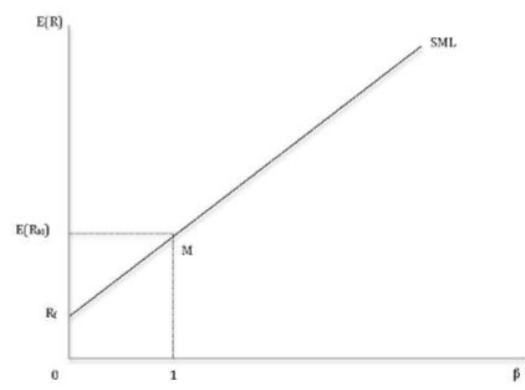
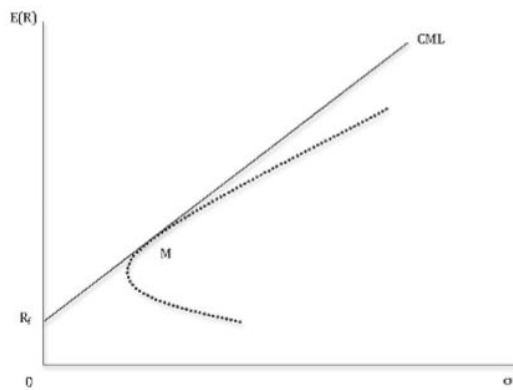
- A zero  $\beta$  asset carries no market risk: earns the risk-free rate ( $R_f$ )
- An asset with a  $\beta$  above one is said to be aggressive
  - Moves more than the market if the market moves
- An asset with a  $\beta$  between 0 and 1 is said to be defensive
  - Moves less than the market if the market moves
- Beta can also be negative
  - Rare

## Capital market line & Security market line

**The capital market line (CML)** shows the highest possible combination of risk and return when you are allowed to combine risk-free assets and the market portfolio

- Where the only thing that varies is the weight between the market portfolio and the risk-free asset

**The Security market line (SML)**, this is the plot of  $\beta$  versus expected return, according to CAPM all securities should lie on this line



## Deviations from SML

The CAPM tells us the return that a stock should earn based on its risk, so if a stock is above the SML

- That stock has a higher expected return than the CAPM says it should have.
  - **Abnormal or excess positive return**
  - We call this deviation alpha, meaning the stock is currently under-priced.
- Trading implication: buy the stock and give it a higher weight (buy the stock)
- You earn expected return without bearing the appropriate level of risk

# Finance 1 – IBEB – Lecture and Exercise lecture 5, week 5

## Cost of capital

Resources are not obtained by a firm for free, because this amount could have been used to fund other projects. Thus, the opportunity cost of using this resource is equal to the value it would have when using the resource for the best possible alternative. The cost of capital of an investment is the expected return of available investments with the same beta.

- Debt cost of capital
- Equity cost of capital
- Project cost of capital

## Equity cost of capital

The CAPM can be used to estimate cost of capital this estimate is provided by the Security Market Line (SML) equation:

$$r_i = r_f + \beta_i \times (E(R_{mkt}) - r_f)$$

The cost of capital of any investment opportunity equals the expected return of available investments with the same beta, to estimate the formula we need an estimate of:

- The risk free rate
- The expected return on the market
- The beta

## The Market Portfolio

The market portfolio is almost impossible to define because there are just too many assets which are illiquid

- The S&P 500 is used the most as it's the most common with large amounts of data, but it only reflects about 10% of global wealth

- If you want to invest in the S&P 500 and hold it as the market portfolio, then in theory you should hold a value weighted portfolio of all stock that comprise the S&P 500

**Value-Weighted Portfolio** – a portfolio that consists of securities that are held in proportion to their market capitalization.

Additionally, it is also a **Passive portfolio** that does not require rebalancing even if market prices change.

In a portfolio like the market portfolio the investment in each security  $i$  is proportional to its market capitalization:

$$MV_i = (\text{Number of Shares of } i \text{ Outstanding}) \cdot (\text{Price of } i \text{ per share}) = N_i \cdot P_i$$

The weight of each security in the portfolio can be calculated as follows:

$$x_i = \frac{\text{Market Value of } i}{\text{Total Market Value of all securities}} = \frac{MV_i}{\sum_j MV_j}$$

## The Market Risk Premium

We need to determine the risk-free rate

- The yield on US Treasury securities is most commonly used due to it's liquidity
- Using mostly 10-to-30-year treasuries

The risk premium  $(E(R_{mkt}) - r_f)$  on the market can be estimated using the **historical** average excess return of the market over the risk-free interest rate however, there are **drawbacks** for using historical data.

- Standard errors of estimates are large, or data sampling issues, causing returns difficult to measure
- It is backward-looking and, hence, may not be representative of current expectations

A **possible alternative** could be to look at what is implied by the market and solve for the discount rate:

$$r_{mkt} = \frac{div_1}{P_0} + g = \text{Dividend yield} + \text{Expected dividend growth rate}$$

## Beta estimation

**Beta** is the expected percent change in the excess return of the security for a 1% change in the excess return of the market portfolio

Linear regression can be applied to estimate the excess return on stock  $i$  (dependent variable) with market excess return (independent variable)

- Due to measurement error, beta's are easier to estimate for (industry) portfolios than individual returns

$$(R_i - r_f) = \alpha_i + \beta_i(R_{mkt} - r_f) + \varepsilon_i$$

Where  $\alpha_i$  is the intercept term,  $\beta_i(R_{mkt} - r_f)$  is the sensitivity of the stock to market risk and  $\varepsilon_i$  is the error term (zero on average)

Essentially,  $\alpha_i$  is the measure of historical return on stock against the estimation of SML.

- Positive alpha means that historical return on stock was better than predicted by CAPM
- Negative alpha means that historical return was below SML.

## Debt cost of capital

Debt betas are hard to estimate because many bonds trade infrequently

**Yield to maturity** is the IRR an investor will earn from holding the bond to maturity and receiving its promised payments.

- If there is little risk the firm will default, yield to maturity is a reasonable estimate of investors' expected rate of return.
- If there is significant risk of default, yield to maturity will overstate investors' expected return.

Consider a one-year bond with YTM of  $y$

- For each \$1 invested in the bond today, the issuer promises to pay  $\$(1 + y)$  in one year
- Suppose the bond will default with probability  $p$ , in which case bond holders receive only  $\$(1 + y - L)$ , where  $L$  is the expected loss per \$1 of debt in the event of default.

Expected return of the bonds can be calculated using the formula:

$$r_d = (1 - p)y + p(y - L) = y - pL$$

$$= \text{Yield to Maturity} - \text{probability of default} \times \text{Expected loss rate}$$

### Example – using expected default rate

Consider a firm with a 4 year ‘junk’ bond with a CCC rating. The yield to maturity is 12%. The risk-free rate is 5% and the risk premium is 7%.

- We are not in a crisis
- Expected loss rate of 60%

Rating:	AAA	AA	A	BBB	BB	B	CCC	CC-C
<b>Default Rate:</b>								
Average	0.0%	0.1%	0.2%	0.5%	2.2%	5.5%	12.2%	14.1%
In Recessions	0.0%	1.0%	3.0%	3.0%	8.0%	16.0%	48.0%	79.0%

Compute the expected return on this debt, using the average historical default rates by rating from the table

- $y - pL$ , where  $p$  is 12.2%
- $0.12 - (0.122 \cdot 0.6) = 4.68\%$

### Example – using average betas

Consider a firm with a 4 year ‘junk’ bond with a CCC rating. The yield to maturity is 12%. The risk-free rate is 5% and the risk premium is 7%.

- Expected loss rate of 60%

Rating:	A or more	BBB	BB	B	CCC
<b>Debt beta</b>	<0.05	0.10	0.17	0.26	0.31

This time we apply the CAPM

- Risk-free + (beta · YTM)
- $0.05 + 0.31 \cdot 0.07 = 7.17\%$

## Project's cost of capital

### All-equity comparables

To measure the compensation for the risk of a new project when it is fully equity financed, but different from the average project in the company, we want to find an all-equity financed firm with similar business operations to a new project for comparison.



By using comparable firm's beta and cost of capital we can estimate the cost of capital for the new project.

- If it is not possible to find an all-equity financed firm we can also make estimates from a levered firm (financed by both debt and equity) with similar business activities.
- As a result of having debt financing the return on equity will be higher due to higher risk. Hence we want to use the unlevered cost of capital.

The **unlevered cost of capital** – the expected return required by investors to hold the firm's underlying

The weighted average of the firm's equity and debt cost of capital can be calculated using the formula:

$$r_u = \frac{E}{E + D} \times r_E + \frac{D}{E + D} \times r_D$$

**Note:** "perfect" world without taxes and frictions assumed

Similarly: Unlevered Beta

$$\beta_u = \frac{E}{E + D} \times \beta_E + \frac{D}{E + D} \times \beta_D$$

When calculating  $\frac{D}{E+D}$  ratio it is important to use the net debt. This is because cash is a risk-free asset that reduces the average risk of a firm's assets.

$$\text{Net Debt} = \text{Debt} - \text{Excess cash} - \text{Short term investments}$$

**Operating Leverage** is the proportion of fixed to variable costs of the project.

- A higher proportion of fixed costs implies higher sensitivity of the project's cash flows to market risks, meaning higher beta and cost of capital.

## The Weighted Average Cost of Capital (WACC)

When the assumption of the "perfect" world is relaxed, we need to account for taxes.

$$\text{Effective after-tax interest rate} = r(1 - \tau_c)$$

The Weighted Average Cost of Capital is then equal to:

$$r_{WACC} = \frac{E}{E + D} \times r_E + \frac{D}{E + D} \times r_D \times (1 - \tau_c)$$

Given a target leverage ratio:

$$r_{WACC} = r_u - \frac{D}{E + D} \times \tau_c r_D$$

Where  $r_u$  is the unlevered cost of capital or pre-tax WACC.

## WACC and pre-tax WACC

Unlevered cost of capital shows the expected return on holding a firm's assets, which in the real world with taxes can be used for evaluation of all-equity projects with the same risk as the firm.

- Taking into account taxes, WACC adjusts for the capital structure of the firm.

# Investor behavior and capital efficiency

## Competition and Capital markets

**Stock's alpha** - the difference between a stock's expected return and its required return according to the security market line (CAPM return). When the market portfolio is efficient all stocks are on SML and alpha is equal to zero.

Stock's alpha:  $\alpha_s = E[R_s] - r_s$

Security market line:  $r_s = r_f + \beta_s \times (E[R_{mkt}] - r_f)$

## Deviation from SML

Stock prices adjust to news

- Positive alpha (=higher expected return than CAPM predicts)
- Investors buy at a lower price than CAPM prediction until expected returns go down again
- The stock is back on SML

## Information and rational expectation

Informed Vs. Uninformed Investors

- In the CAPM framework, investors should hold the market portfolio combined with risk-free investments.
- An uninformed investor can buy an ETF and T-bills

- Provides an alpha of zero (if market is efficient)
- Prices should respond to information

The market portfolio can be inefficient if

- Information was misinterpreted by a substantial number of investors who believe they are earning a positive alpha when they are actually earning a negative alpha
- A significant number of investors are willingly hold inefficient portfolio because they also care about other aspects of portfolios other than returns and volatility (like ESG)

## Systematic trading biases

Biases become problematic when they occur systematically instead of randomly. Some common biases are:

- **Disposition effect**- tendency to hold losing stocks for too long, as investors do not want to realize losses
- **Herd behavior**- occurs when individuals start actively following each other

## Multiple factor model

CAPM used so far is an example of a **single-factor model** which assumes that the only risk is market risk, however there is a debate whether markets are truly efficient.

3 main patterns have been observed which is debated on whether they are just temporary or actually persistent

- Small market capitalization stocks have historically earned higher average returns than the market portfolio, even after accounting for their higher beta. **"Small Cap" stocks**
- High book-to-market stocks have historically earned higher average returns than low book-to-market stocks. **"Value" stocks**
- Buy stocks with high-past returns and short stocks with past low returns.

### **Momentum**

**Multi-factor model of risk** is used where there are other factors that explain returns, hence, CAPM proposed portfolios may not be efficient. In this case, to

construct efficient portfolios we need to use factor portfolios that are covering different risk factors.

Multifactor model of risk for given N factor portfolios:

$$E[R_S] = r_f + \beta_S^{F1} \times (E[R_{F1}] - r_f) + \beta_S^{F2} \times (E[R_{F2}] - r_f) + \dots + \beta_S^{FN} \times (E[R_{FN}] - r_f) = r_f + \sum_{n=1}^N \beta_S^{Fn} \times (E[R_{Fn}] - r_f)$$

The **Arbitrage Pricing Theory (APT)** is an example of a multifactor model

## Self-financing portfolio

A **self-financing portfolio** can be constructed by going short in some stocks and long in others with the same market value.

For self-financing factor portfolios the formula comes down to (as the risk free rate drops out):

$$E[R_S] = r_f + \beta_S^{F1} \times E[R_{F1}] + \beta_S^{F2} \times E[R_{F2}] + \dots + \beta_S^{FN} \times E[R_{Fn}] = r_f + \sum_{n=1}^N \beta_S^{Fn} \times (E[R_{Fn}])$$

The Fama-French-Carhart (FFC) Factor Specifications was created to try and predict stock returns better than CAPM by incorporating other risks by creating a self-financing portfolio based on patterns we noted

- Buy small-cap stocks, short large cap stocks: **small-minus-big portfolio (SMB)**
- Buy high book-to-market (value) stocks, short low book-to-market stocks (growth): **high-minus-low portfolio (HML)**
- Buy stocks with high one-year past returns, short stocks with low past returns: **prior one-year momentum portfolio (PRIYR)**

$$E[R_S] = r_f + \beta_S^{Mkt} \times (E[R_{Mkt}] - r_f) + \beta_S^{SMB} \times (E[R_{SMB}] - r_f) + \beta_S^{HML} \times (E[R_{HML}] - r_f) + \beta_S^{PRIYR} \times (E[R_{PRIYR}] - r_f)$$

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